Recent progress for semi-linear damped σ-evolution models

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Abstract We discuss the Cauchy problem for semi-linear damped σ-evolution models

$$u_{tt} + (-\Delta)^\sigma u + b(t)(-\Delta)^\delta u_t = f(u, u_t, |D|^a u), \ \ u(0,x) = u_0(x), \ \ u_t(0,x) = u_1(x)$$

with different model power non-linearities, $a \in (0, \sigma]$. Our main issue is to determine the critical exponent dividing the range of admissible exponents into those producing, in general, a blow-up behavior of solutions and those allow the proof of global existence (in time) of small data solutions. Matsumura type estimates for solutions to parameter-dependent Cauchy problems are an important tool in our approach. We will explain how modern results of harmonic analysis can be used to treat the non-linear terms. Some discussion of optimality of our results and some open problems complete the talk.

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