Timetable of the session "Dispersive Equations"
Organizers: F. Hirosawa (Yamaguchi) and M. Reissig (Freiberg)

Time	Wednesday, July 15	Thursday, July 16	Friday, July 17
09.00 - 09.45	Gobbino	Yordanov	Matsuyama
09.45 - 10.30	Ghisi	Karp	Kubo
13.45 - 14.30	Mochizuki	Nishitani	Fang
14.30 - 15.00	Georgiev	Ebert	Saito
15.00 - 15.45	Del Santo	Colombini	Suzuki
16.00 - 16.30	Pivetta	Galstyan	Choi
16.30 - 17.00	Hirosawa	Herrmann	Reissig
17.00 - 17.30	Uesaka	Jung	D'Abbicco
17.30 - 18.15	Nakazawa	Yagdjian	Picard

Wednesday, July 15 Chair: Reissig

Opening address by M. Reissig

09.00 - 09.45: M. Gobbino (Pisa, Italy)
Existence and uniqueness results for Kirchhoff equations in Gevrey type spaces

09.45 - 10.30: M. Ghisi (Pisa, Italy)
Hyperbolic - parabolic singular perturbations for Kirchhoff equations

Chair: Hirosawa

13.45-14.30: K. Mochizuki (Tokyo, Japan)
Uniform resolvent estimates and smoothing effects for magnetic Schrödinger operators

14.30-15.00: V. Georgiev (Pisa, Italy)
Stability of solitary waves for Hartree type equation

15.00 - 15.45: D. Del Santo (Trieste, Italy)
Continuous dependence for backward parabolic operators with Log-Lipschitz coefficients

Chair: Reissig

16.00 - 16.30: M. Pivetta (Trieste, Italy)
Backward uniqueness for the system of thermoelastic waves with non Lipschitz-continuous coefficients

16.30 - 17.00: F. Hirosawa (Yamaguchi, Japan)
Wave equations with time dependent coefficients

17.00 - 17.30: H. Uesaka (Tokyo, Japan)
Blow-up and a blow-up boundary for a semilinear wave equation with some convolution nonlinearity

17.30 - 18.15: H. Nakazawa (Chiba, Japan)
Decay and scattering for wave equations with dissipations in layered media

Thursday, July 16 Chair: Reissig

09.00 - 09.45: B. Yordanov (Tennessee-Knoxville, USA)
Global existence in Sobolev spaces for a class of nonlinear Kirchhoff equations

09.45 - 10.30: L. Karp (Karmiel, Israel)
On the well-posedness of the vacuum Einstein's equations

Chair: Hirosawa

13.45 - 14.30: T. Nishitani (Osaka, Japan)
On the Cauchy problem for non-effectively hyperbolic operators, the Gevrey 4 well-posedness

14.30 - 15.00: M.R. Ebert (Sao Paulo, Brazil)
On the loss of regularity for a class of weakly hyperbolic operators

15.00 - 15.45: F. Colombini (Pisa, Italy)
Local solvability beyond condition PSIKaren Yagdjian (UTPA,USA)
Fundamental solutions for hyperbolic operators with variable coefficients

Chair: Reissig

16.00 - 16.30: A. Galstyan (Edinburg, USA)
Wave equation in Einstein de Sitter spacetime

16.30 - 17.00: T. Herrmann (Freiberg, Germany)
Precise loss of derivatives for evolution type models

17.00 - 17.30: T. Jung (Kunsan, South Korea)
Critical point theory applied to a class of systems of superquadratic wave equations

17.30 - 18.15: Karen Yagdjian (UTPA,USA)
Fundamental solutions for hyperbolic operators with variable coefficients

Friday, July 17 Chair: Hirosawa

09.00 - 09.45: T. Matsuyama (Hiratsuka, Japan)
Strichartz estimates for hyperbolic equations in an exterior domain

09.45 - 10.30: H. Kubo (Tohoku, Japan)
Generalized wave operator for a system of wave equations

Chair: Reissig

13.45 - 14.30: D. Fang (Hangzhou, P.R.China)
Zakharov system in infinite energy space

14.30 - 15.00: J. Saito (Tokyo,Japan)
The Boussinesq equations based on the hydrostatic approximation

15.00 - 15.45: R. Suzuki (London, UK)
Blow-up of solutions of a quasilinear parabolic equation

Chair: Hirosawa

16.00 - 16.30: Q-H. Choi (Incheon, South Korea)
Multiple solutions for nonlinear parabolic systems

16.30 - 17.00: M. Reissig (Freiberg, Germany)
The Log-effect for 2 by 2 hyperbolic systems

17.00 - 17.30: M. D'Abbicco (Bari, Italy)
$L^p - L^q$ Estimates for hyperbolic systems

17.30 - 18.15: R. Picard (Dresden, Germany)
On a model for consolidation of poro-elastic media

Closing address by F. Hirosawa.

$L^p - L^q$ ESTIMATES FOR HYPERBOLIC SYSTEMS

M. D'Abbicco (Bari, Italy)
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We present $L^p - L^q$ estimates for the solution of some strictly hyperbolic first order systems with bounded time dependent coefficients. In the equation setting, Reissig and others obtained such estimates by using WKB representation of the solutions. Here the crucial point is to find analogous assumptions on the coefficients of the system so that this approach still works. In particular an application of this result cover some known estimates for scalar equations.

This is a joint work with Sandra Lucente and Giovanni Taglialatela from University of Bari.

M. D'Abbicco, S. Lucente, G. Taglialatela, *L^p-L^q estimates for regularly hyperbolic systems*, submitted.

MULTIPLE SOLUTIONS FOR NONLINEAR PARABOLIC SYSTEMS

Q-H. Choi (Incheon, South Korea)
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We have a concern with the existence of solutions (ξ, η) for perturbations of the parabolic system with Dirichlet boundary condition

$$\begin{aligned} \xi_t &= -L\xi + \mu g(3\xi + \eta) - s\phi_1 - h_1(x,t) \quad \text{in } \Omega \times (0, 2\pi), \\ \eta_t &= -L\eta + \nu g(3\xi + \eta) - s\phi_1 - h_2(x,t) \quad \text{in } \Omega \times (0, 2\pi). \end{aligned} \tag{0.1}$$

We prove a uniqueness theorem when the nonlinearity does not cross eigenvalues. We also investigate multiple solutions $(\xi(x,t), \eta(x,t))$ for perturbations of the parabolic system with Dirichlet boundary condition when the nonlinearity f' is bounded and $f'(-\infty) < \lambda_1, \lambda_n < (3\mu + \nu) f'(+\infty) < \lambda_{n+1}$.

LOCAL SOLVABILITY BEYOND CONDITION PSI

F. Colombini (Pisa, Italy)
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It is well known that condition P (PSI) is necessary and sufficient in order to have local solvability for differential (pseudo-differential) operators of principal type with coefficients sufficiently regular.

We study some cases when such conditions are not satisfied.

These are two joint papers with Ludovico Pernazza and François Treves and with Paulo Cordaro and Ludovico Pernazza.

CONTINUOUS DEPENDENCE FOR BACKWARD PARABOLIC OPERATORS WITH LOG-LIPSCHITZ COEFFICIENTS

D. Del Santo (Trieste, Italy)
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We consider the following backward parabolic equation

$$\partial_t u + \sum_{i,j} \partial_{x_i}(a_{i,j}(t,x)\partial_{x_j} u) + \sum_j b_j(t,x)\partial_{x_j} u + c(t,x)u = 0 \tag{0.2}$$

on the strip $[0,T] \times \mathbb{R}^n \ni (t,x)$. We suppose that

- for all $(t,x) \in [0,T] \times \mathbb{R}^n$ and for all $i,j = 1 \dots n$,
$$a_{i,j}(t,x) = a_{j,i}(t,x);$$
- there exists $k > 0$ such that, for all $(t,x,\xi) \in [0,T] \times \mathbb{R}^n \times \mathbb{R}^n$,
$$k|\xi|^2 \leq \sum_{i,j} a_{i,j}(t,x)\xi_i\xi_j \leq k^{-1}|\xi|^2;$$
- for all $i,j = 1,\dots,n$, $a_{i,j} \in \mathrm{LL}([0,T], L^\infty(\mathbb{R}^n)) \cap L^\infty([0,T], C_b^2(\mathbb{R}^n))$ and b_j, $c \in L^\infty([0,T], C_b^2(\mathbb{R}^n))$,

(where $a \in \mathrm{LL}([0,T], L^\infty(\mathbb{R}^n))$ means that the function a is Log–Lipschitz–continuous with respect to time with values in L^∞, i.e.

$$\sup\left\{\frac{\|a(t,\cdot) - a_{i,j}(s,\cdot)\|_{L^\infty(\mathbb{R}^n)}}{|t-s|(1+|\log|t-s||)} \mid t,s \in [0,T],\ 0 < |t-s| \leq 1\right\} \leq +\infty).$$

Let $\mathcal{E} := C^0([0,T], L^2(\mathbb{R}^n)) \cap C^0([0,T[, H^1(\mathbb{R}^n)) \cap C^1([0,T[, L^2(\mathbb{R}^n))$.

Our first achievement is a continuous dependence result which is valid only for sufficiently small t.

Theorem 1. *There exist* $\sigma \in]0,T[$*, and for all* $\bar{\sigma} \in]0,\sigma/4[$ *there exist* ρ*,* $\bar{M}$*,* N*,* $\delta > 0$ *such that, if* $u \in \mathcal{E}$ *is a solution of the equation (0.2) with* $\|u(0,\cdot)\|_{L^2} \leq \rho$*, then*

$$\sup_{t\in[0,\bar{\sigma}]} \|u(t,\cdot)\|_{L^2} \leq \bar{M}(1+\|u(\sigma,\cdot)\|_{L^2})e^{-N(|\log\|u(0,\cdot)\|_{L^2}|)^{\delta}}. \tag{0.3}$$

In Theorem 1, $\sigma = \min\{T, 1/\alpha_1\}$, where α_1 depends only on the constant k, on the LL norms of the a_{ij}'s and on the L^∞ norms of the coefficients and its derivatives. This allows one to iterate the local result of Theorem 1 a finite number of times, so as to obtain the following global continuous dependence result.

Theorem 2. *For all* $T' \in]0,T[$ *and for all* $D > 0$ *there exist* ρ'*,* M'*,* N'*,* $\delta' > 0$ *such that if* $u \in \mathcal{E}$ *is a solution of the equation (0.2) with* $\sup_{t\in[0,T]} \|u(t,\cdot)\|_{L^2} \leq D$ *and* $\|u(0,\cdot)\|_{L^2} \leq \rho'$*, then*

$$\sup_{t\in[0,T']} \|u(t,\cdot)\|_{L^2} \leq M' e^{-N'|\log\|u(0,\cdot)\|_{L^2}|^{\delta'}}. \tag{0.4}$$

This is a joint work with Martino Prizzi from University of Trieste.

Precise statements, remarks and proofs can be found at the address http://arxiv.org/abs/0801.3990.

ON THE LOSS OF REGULARITY FOR A CLASS OF WEAKLY HYPERBOLIC OPERATORS

M.R. Ebert (Sao Paulo, Brazil)
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In this work we consider the Cauchy problem

$$\begin{aligned} Pu &= \partial_t^2 u - \lambda^2(t)\sum_{i,j=1}^{n} a_{ij}(t)\partial^2_{x_i x_j} u + \lambda(t)\sum_{i=1}^{n} c_i(t)\partial^2_{t x_i} u \\ &= f(x,t,u,\partial_t u, \lambda^{'}(t)\nabla_x u), \end{aligned} \tag{0.5}$$

$$u(x,0) = u_0(x),\ \partial_t u(x,0) = u_1(x), \tag{0.6}$$

where P is weakly hyperbolic in a neighborhood of $\{t = 0\}$, that is,

$$\text{the roots of } p(x,t,\xi,\tau) \text{ in } \ \tau \text{ are real;} \tag{0.7}$$

here $p = p(x,t,\xi,\tau)$ is the principal symbol of P. Examples show that, differently to the strictly hyperbolic case, under (0.5), (0.6) and (0.7) the solution might not exist. In addition to condition (0.7), various authors presented sufficient conditions, usually called Levi conditions, for the Cauchy problem to be well posed in Sobolev spaces. Those type of conditions relate p with lower order terms of P. In this work, we narrowed the bounds for the optimal Sobolev loss of regularity under some sharp Levi conditions.

This work was done in collaboration with Rafael A. dos Santos Kapp and Jose Ruidival dos Santos Filho, both from UFSCar (Brazil).

ZAKHAROV SYSTEM IN INFINITE ENERGY SPACE

D. Fang (Hangzhou, P.R.China)
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We consider the Zakharov system in space dimension two. We will show a L^2-concentration result for the data without finite energy, when blow-up of the solution happens, and a low regularity global well-posedness result. The proof uses a refined I-method originally initiated by Colliander, Keel, Staffilani, Takaoka and Tao. A polynomial growth bound for the solution is also given.

This talk is based on some joint works with Sijia Zhong and Hartmut Pecher.

WAVE EQUATION IN EINSTEIN DE SITTER SPACETIME

A. Galstyan (Edinburg, USA)
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In this talk we introduce the fundamental solutions of the wave equation in the Einstein-de Sitter spacetime. The last one describes the simplest non-empty expanding model of the universe. The covariant d'Alembert's operator in the Einstein-de Sitter spacetime belongs to the family of the non-Fuchsian partial differential operators. In this talk we investigate initial value problem for this equation and give the explicit representation formulas for the solutions.

The equation is strictly hyperbolic in the domain with positive time. On the initial hypersurface its coefficients have singularities that make difficulties in studying of the initial value problem. In particular, one cannot anticipate the well-posedness in the Cauchy problem for the wave equation in the Einstein-de Sitter spacetime. The initial conditions must be modified to so-called weighted initial conditions in order to adjust them to the equation.

We also show the $L_p - L_q$ estimates for solutions. Thus, we have prepared all necessary tools in order to study the solvability of semilinear wave equation in the Einstein-de Sitter spacetime.

This is a joint work with Tamotu Kinoshita (University of Tsukuba, Japan) and Karen Yagdjian (UTPA,USA).

STABILITY OF SOLITARY WAVES FOR HARTREE TYPE EQUATION

V. Georgiev (Pisa, Italy)
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The goal of this talk is to present some dispersive estimates appearing in the study of the nonlinear Hartree type equation with external Coulomb potential

$$\frac{1}{2}\Delta\chi + \omega\chi = e^2\left(q(|\chi|^2) - \frac{Z}{|x|}\right)\chi, \quad x \in \mathbf{R}^3, \quad \int_{\mathbf{R}^3} \chi^2 = N, \tag{0.8}$$

where

$$q(f)(x) = \int_{\mathbf{R}^3} \frac{f(y)}{|x-y|}\, dy. \tag{0.9}$$

Under neutrality condition $N = Z$ solitary waves exist and a linearizing the nonlinear problem around solitary solution one derives a linear perturbation of the Schrödinger equation with non - selfadjoint matrix. Suitable diagonalization procedure enables one to introduce appropriate modified generators of the pseudoconformal group and study the dispersive properties of the corresponding linear problem.

This is a joint work with George Venkov (Technical University of Sofia, Bulgaria) and Jimmy Alfonso Mauro (University of Pisa, Italy).

HYPERBOLIC - PARABOLIC SINGULAR PERTURBATIONS FOR KIRCHHOFF EQUATIONS

M. Ghisi (Pisa, Italy)
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We consider the second order Cauchy problem

$$\epsilon u'' + g(t)u' + m(|A^{1/2}u|^2)Au = 0, \quad u(0) = u_0, u'(0) = u_1,$$

where $\epsilon > 0$, g is a positive function, m is a non-negative C^1 function, A is a self-adjoint non-negative operator with dense domain $D(A)$ in a Hilbert space, and $(u_0, u_1) \in D(A) \times D(A^{1/2})$. We prove the global solvability of the Cauchy problem under different conditions on the functions m and g, including the case where $m(0) = 0$, and the case where $g(t)$ tends to 0 as t tends to +infinity (weak dissipation). We also consider the behavior of solutions as t tends to +infinity (decay estimates), and as ϵ tends to 0.

EXISTENCE AND UNIQUENESS RESULTS FOR KIRCHHOFF EQUATIONS IN GEVREY TYPE SPACES

M. Gobbino (Pisa, Italy)
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We consider the second order Cauchy problem

$$u'' + m(|A^{1/2}u|^2)Au = 0, \qquad u(0) = u_0, \quad u'(0) = u_1,$$

where $m : [0, +\infty) \to [0, +\infty)$ is a continuous function, and A is a self-adjoint nonnegative operator with dense domain on a Hilbert space.

In this conference we present three results.

- The first result is local existence for initial data in suitable spaces depending on the continuity modulus of the nonlinear term m. These spaces are a natural generalization of Gevrey spaces to the abstract setting. We also show that solutions with less regular data may exhibit an instantaneous derivative loss.
- The second result concerns uniqueness in the case where the nonlinear term is not Lipschitz continuous.
- The last result concerns the global solvability. Roughly speaking, we show that every initial datum in the spaces where local solutions exist is the sum of two initial data for which the solution is actually global.

PRECISE LOSS OF DERIVATIVES FOR EVOLUTION TYPE MODELS

T. Herrmann (Freiberg, Germany)
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The goal of this talk is to present statements about well-posedness for Cauchy problems for degenerate p-evolution equations with time-dependent coefficients. Degeneracy means that the p-evolution operators may have characteristics of variable multiplicity. On the one hand we are interested to apply phase space analysis to derive results for well-posedness with a (possible) loss of regularity. On the other hand we discuss strategies how to show optimality of the results and sharpness of the assumptions. Here Floquet theory and instability arguments form the core of our strategies. We distinguish between optimality for the leading coefficients of the principal part and for coefficients of the remaining principal part.

WAVE EQUATIONS WITH TIME DEPENDENT COEFFICIENTS

F. Hirosawa (Yamaguchi, Japan)
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We consider the following Cauchy problem for a wave equation with time dependent propagation speed $a = a(t)$:

$$\begin{cases} (\partial_t^2 - a(t)^2\Delta)u = 0, & (t,x) \in (0,\infty) \times \mathbb{R}^n, \\ (u(0,x), (\partial_t u)(0,x)) = (u_0(x), u_1(x)), & x \in \mathbb{R}^n, \end{cases} \tag{1}$$

where $a \in C^1([0,\infty))$ and $0 < a_0 \le a(t) \le a_1$. Then the total energy of (1) at t is given by

$$E(t) = \frac{1}{2}\left(a(t)^2\|\nabla u(t,\cdot)\|_{L^2}^2 + \|\partial_t u(t,\cdot)\|_{L^2}^2\right).$$

If $a(t)$ is a constant, then the energy conservation $E(t) \equiv E(0)$ holds. However, such a property does not hold in general for variable variable propagation speeds; thus we consider the following energy estimates:

$$e^{-\eta(t)}E(0) \le E(t) \le e^{\eta(t)}E(t) \quad (t \to \infty), \tag{2}$$

where $\eta(t)$ is monotone increasing and satisfies $\eta(t) > 0$.

If $|a'(t)| \le C(1+t)^{-\beta}$ for $\beta \ge 0$, then the estimates (2) hold with $\eta(t) = Ct^{-\beta+1}$ for $\beta < 1$ and $\eta(t) = C$ for $\beta > 1$; thus faster decaying $|a'(t)|$ contribute to the stabilization of the energy. Moreover, if $a \in C^m([0,\infty))$ $(m \ge 2)$, then the order of $\eta(t)$ can be improved corresponding to m under the following assumption, which is called the *stabilization property*:

$$\int_0^t |a(s) - a_\infty|\,ds = O(t^\alpha) \quad (0 \le \alpha < 1), \quad a_\infty = \lim_{t\to\infty}\frac{\int_0^t a(s)\,ds}{t}. \tag{3}$$

Theorem 1([1]) *If $a \in C^m([0,\infty))$ $(m \ge 2)$ satisfies* (3) *for a $\alpha \in [0,1)$ and*

$$|a^{(k)}(t)| \le C_k(1+t)^{-k\beta} \quad (k = 1,\cdots,m), \tag{4}$$

then (2) *hold with $\eta(t) = Ct^{\sigma_m}$, where $\sigma_m = \max\{0, \alpha - \beta + (1-\alpha)/m\}$.*

Let us consider the limit case of $m \to \infty$ to introduce the Gevrey class γ^ν:

$$\gamma^\nu = \left\{f(t) \in C^\infty([0,\infty))\,;\, |f^{(k)}(t)| \le C\rho^{-k}k!^\nu, \ \exists\rho > 0\right\} \quad (\nu > 1).$$

For $a \in \gamma^\nu$ and a non-negative constant δ we introduce the following conditions:

$$|a^{(k)}(t)| \le Ck!^\nu\left((1+t)^\alpha(\log(e+t))^\delta\right)^{-k} \quad (k = 1,2,\cdots). \tag{5}$$

Then we have the following result, which gives more precise estimates of (2) for $m = \infty$ and $\alpha = \beta$:

Theorem 1([2]) *If* $a \in \gamma^\nu$ *satisfies* (3) *and* (5) *for a* $\alpha \in [0, 1)$, *then* (2) *hold with* $\eta(t) = C(\log t)^\sigma$, *where* $\sigma = \max\{0, \nu - \delta\}$.

Summarizing Theorem 1 and Theorem 2, we have the following table for the relations between the smoothness of $a(t)$ and the order of $\eta(t)$ under the assumptions (3) and (5) with $\delta = 0$:

$a(t)$	C^1	C^m $(m \geq 2)$	C^∞	γ^ν $(\nu > 1)$
$\eta(t)$	$Ct^{1-\alpha}$	$Ct^{\frac{1-\alpha}{m}}$	Ct^ε	$C(\log t)^\nu$

where ε is an arbitrarily positive constant.

References

[1] F. Hirosawa, On the asymptotic behavior of the energy for the wave equations with time depending coefficients. *Math. Ann.* **339** (2007), 819–839.

[2] F. Hirosawa, Energy estimates for wave equations with time dependent propagation speeds of the Gevrey class. Preprint.

CRITICAL POINT THEORY APPLIED TO A CLASS OF SYSTEMS OF SUPERQUADRATIC WAVE EQUATIONS

T. Jung (Kunsan, South Korea)
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We show the existence of a nontrivial solution for a class of systems of superquadratic nonlinear wave equations with Dirichlet boundary conditions and periodic conditions with a superquadratic nonlinear terms at infinity which have continuous derivatives. We approach the variational method and use the critical point theory which is the Linking Theorem for the strongly indefinite corresponding functional.

ON THE WELL-POSEDNESS OF THE VACUUM EINSTEIN'S EQUATIONS

L. Karp (Karmiel, Israel)
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The Cauchy problem of the vacuum Einstein's equations determines a semi-metric $g_{\alpha\beta}$ of a space-time with vanishing Ricci curvature $\mathbf{R}_{\alpha,\beta}$ and prescribe initial data.

Under harmonic gauge condition, the equations $\mathbf{R}_{\alpha,\beta}$ are transferred into a system of quasi-linear wave equations which are called *the reduced Einstein equations*. The initial data for Einstein's equations are a proper Riemannian metric h_{ab} and a second fundamental form K_{ab}. However, these data must satisfy Einstein constraint equations and therefore the pair (h_{ab}, K_{ab}) cannot serve as initial data for the reduced Einstein equations.

Previous results in the case of asymptotically flat spacetimes provide a solution to the constraint equations in one type of Sobolev spaces, while initial data for the evolution equations belong to a different type of Sobolev spaces. The aim of our work is to resolve this incompatibility and to show well-posedness of the reduced Einstein vacuum equations in one type of Sobolev spaces.

GENERALIZED WAVE OPERATOR FOR A SYSTEM OF NONLINEAR WAVE EQUATIONS

H. Kubo (Tohoku, Japan)
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In this talk we discuss the asymptotic behavior of solutions to a system of nonlinear wave equations whose decaying rate is actually slower than that of the free solutions. Despite of that fact, we are able to construct wave operators in a generalized sense. The proof is done by finding a nice approximation and introducing a suitable metric (that is not a norm in fact). Moreover, the scattering operators are defined in a generalized sense.

STRICHARTZ ESTIMATES FOR HYPERBOLIC EQUATIONS IN AN EXTERIOR DOMAIN

T. Matsuyama (Hiratsuka, Japan)
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In this talk we will present Strichartz estimates for hyperbolic equations in an exterior domain outside a star-shaped obstacle. This is a joint work with S. Murai (Tokai University).

UNIFORM RESOLVENT ESTIMATES AND SMOOTHING EFFECTS FOR MAGNETIC SCHRÖDINGER OPERATORS

K. Mochizuki (Tokyo, Japan)
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Uniform resolvent estimates for magnetic Schrödinger operators in an exterior domain are obtained under smallness conditions on the magnetic fields and scalar potentials. The results are then used to obtain space-time "L^2"-estimates for the corresponding Schrödinger, Klein-Gordon and wave equations.

DECAY AND SCATTERING FOR WAVE EQUATIONS WITH DISSIPATIONS IN LAYERED MEDIA

H. Nakazawa (Chiba, Japan)
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We consider wave equations with linear dissipations in some layered regions;

$$w_{tt}(x,y,t) - \Delta w(x,y,t) + b(x,y)w_t(x,y,t) = 0, \quad (x,y,t) \in \mathbb{R}^N \times [0,\pi] \times (0,\infty)$$

with Dirichlet boundary conditions

$$w(x,0,t) = w(x,\pi,t) = 0, \quad (x,t) \in \mathbb{R}^N \times (0,\infty).$$

For long-range type of dissipations, e.g., $b_0(1+|x|)^{-1} \le b(x,y) \le b_1$ in $\mathbb{R}^N \times [0,\pi]$ for some $b_0, b_1 > 0$, the total energy decays as t goes to infinity. For short-range type of dissipations, e.g., $0 \le b(x,y) \le b_2(1+|x|)^{-1-\delta}$ in $\mathbb{R}^N \times [0,\pi]$ for some $b_2 > 0$ and $\delta > 0$, scattering solution exists.

Although the proof for scattering is based on Kato's smooth perturbation theory, the singular points called thresholds in the spectrum cause to difficulties.

To eliminate this, density argument using some approximate operators are employed.

This is joint work with Mitsuteru Kadowaki (Ehime University) and Kazuo Watanabe (Gakushuin University).

ON THE CAUCHY PROBLEM FOR NON-EFFECTIVELY HYPERBOLIC OPERATORS, THE GEVREY 4 WELL-POSEDNESS

T. Nishitani (Osaka, Japan)
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The Cauchy problem for non-effectively hyperbolic operators is discussed in the Gevrey classes. Our operators belong to the class of non-effectively hyperbolic operators with symbols vanishing of order 2 on a smooth submanifold of codimension 3 on which the canonical symplectic form has a constant rank. Assuming that there is no null bicharacteristic issuing from a simple characteristic point and landing tangentially on the double characteristic manifold, we prove that the Cauchy problem is Gevrey s well-posed for any lower order term whenever $1 \le s < 4$.

ON A MODEL FOR CONSOLIDATION OF PORO-ELASTIC MEDIA

R. Picard (Dresden, Germany)
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A modification of the material law associated with the well-known Biot system as suggested by M. A. Murad and J. H. Cushman in 1996 describing the consolidation of clays and first rigorously investigated by R. E. Showalter in 2000 is re-considered in the light of a new approach to a comprehensive class of evolutionary problems and extended to large class of anisotropic inhomogeneous media.

The paper is joint work with Des McGhee (Strathclyde University, Glasgow, U.K.).

D. McGhee and R. Picard, *A note on poro-elastic media*, submitted.

BACKWARD UNIQUENESS FOR THE SYSTEM OF THERMOELASTIC WAVES WITH NON LIPSCHITZ-CONTINUOUS COEFFICIENTS

M. Pivetta (Triest, Italy)
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Using the Carleman estimates developed by Koch and Lasiecka [Functional analysis and evolution equations, 389-403, Birkhäuser, Basel, 2008] together with an approximation technique in the phase space, a uniqueness result for the backward Cauchy problem is proved for the system of themoelastic waves having coefficients which are in a class of log-Lipschitz-continuous functions.

THE LOG-EFFECT FOR 2 BY 2 HYPERBOLIC SYSTEMS

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In the talk we are interested to explain how to extend the Log-effect from wave equations with time-dependent coefficients to 2 by 2 strictly hyperbolic systems $\partial_t U - A(t)\partial_x U = 0$. From wave models we know that besides oscillations in the coefficients a possible interaction of oscillations has a strong influence on H^∞ well- or ill-posedness. Moreover, the precise loss of derivatives can be proved. In the case of systems the situation is more complicate. Besides the effects of oscillating entries of the matrix $A = A(t)$ and interactions between the entries of A we have to take into consideration the system character itself. We will prove by using tools from phase space analysis results about H^∞ well- or ill-posedness. The precise loss of regularity is of interest. Moreover, we discuss the question if the loss of derivatives does really appear. These considerations base on the interplay between the Ljapunov and energy functional. Finally, we discuss the cone of dependence property for solutions to 2 by 2 systems.

This is a joint work with Tamotu Kinoshita from University of Tsukuba.

T. Kinoshita and M.Reissig, *The Log-effect for 2 by 2 hyperbolic systems*, 29 A4, submitted.

THE BOUSSINESQ EQUATIONS BASED ON THE HYDROSTATIC APPROXIMATION

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The Boussinesq equations are studied in the field of dynamic meteorology. Atmospheric flow in meteorology are described by the Boussinesq equations. Due to the fact that the aspect ratio

$$\varepsilon = \frac{\text{characteristic depth}}{\text{characteristic width}}$$

is very small in most geophysical domains, asymptotic models have been used. One of the models is the hydrostatic approximation of the Boussinesq equations. We consider the Boussinesq equations in the domains with very small aspect ratio and prove a convergence theorem for this model.

BLOW-UP OF SOLUTIONS OF A QUASILINEAR PARABOLIC EQUATION

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We consider nonnegative solutions of the Cauchy problem for quasi-linear parabolic equations $u_t = \Delta u^m + f(u)$, where $m > 1$ and $f(\xi)$ is a positive function in $\xi > 0$ satisfying $f(0) = 0$ and a blow-up condition $\int_1^\infty 1/f(\xi)\,d\xi < \infty$. We study under what conditions on $f(\xi)$ all nontrivial solutions blow up.

This is a joint work with Noriaki Umeda from The University of Tokyo.

BLOW-UP AND A BLOW-UP BOUNDARY FOR A SEMILINEAR WAVE EQUATION WITH SOME CONVOLUTION NONLINEARITY

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Caffarelli and Friedman investigated blow-up boundaries for a Cauchy problem of semi-linear wave equations with power nonlinearity and showed remarkable results (see [1]). The same problem for a system of semi-linear wave equations was treated in [2].

In this talk we consider the following Cauchy problem for a wave equation with convolution nonlinearity:

$$\begin{cases} \Box u = \partial_t^2 u - \Delta u = u^q (V * u^p) \text{ in } \mathbb{R}^3 \times (0, T), \\ u(x, 0) = f(x),\ \partial_t u(x, 0) = g(x) \text{ in } \mathbb{R}^3, \end{cases}$$

where $u^q (V * u^p)(x,t) = u^q(x,t) (\int_{\mathbb{R}^3} \frac{u^p(y,t)}{|x-y|^\gamma} dy)$ with $p, q > 1, 0 < \gamma < 3$.

The blow-up boundary Γ is defined by

$$\Gamma = \partial\{(x,t) : u(x,t) < \infty,\ t > 0\}.$$

We can give several suitable conditions to initial data to show that

1. (1) has a C^2 positive real-valued local solution u,
2. u is monotone increasing in t for any fixed x and, moreover, it satisfies $\partial_t u \geq |\nabla u|$,
3. there exists a positive $T(x)$ for any x such that u keeps its regularity in $\mathbb{R}^3 \times (0, T(x))$ and $\lim_{t \nearrow T(x)} u(x,t) = \infty$.

Then the blow-up boundary Γ exists and is represented by a function $\phi(x) = T(x)$ satisfying

$$|\phi(x) - \phi(y)| \leq |x - y|.$$

References

[1] L.A.Caffarelli and A.Friedman, The blow-up boundary for nonlinear wave equations, *Trans.AMS.* **297** (1986), 223–241.

[2] H.Uesaka, The blow-up boundary for a system of semi-linear wave equations, in: Proceedings of the 6'th ISAAC congress (2009).

FUNDAMENTAL SOLUTIONS FOR HYPERBOLIC OPERATORS WITH VARIABLE COEFFICIENTS

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The goal of this talk is to give a survey of a new approach in the constructing of fundamental solutions for the partial differential operators with variable coefficients and of some resent results obtaining by that approach. This new approach appeals neither to the Fourier transform, nor to the Microlocal Analysis, nor to the WKB-approximation. More precisely, the new integral transformation is suggested which transforms the family of the fundamental solutions of the Cauchy problem for the operator with the constant coefficients without source term to the fundamental solutions for the operators with variable coefficients. The kernel of the transformation is written via Gauss's hypergeometric function.

This approach was successfully applied by the author of this abstract and his coauthors, T.Kinoshita (University of Tsukuba) and A.Galstyan (University of Texas-Pan American), to investigate in the unified way several equations such as the linear and semilinear Tricomi and Tricomi-type equations, Gellerstedt equation, the wave equation in Einstein-de Sitter spacetime, the wave equation and the Klein-Gordon equation in the de Sitter and in the anti-de Sitter spacetime. The listed equations play important role in the gas dynamics, elementary particle physics, quantum field theory in the curved spaces, and cosmology. In particular, for all above mentioned equations, we have obtained representation formulas for the initial-value problem, the $L_p - L_q$-estimates, local and global solutions for the semilinear equations, blow up phenomena, self-similar solutions and number of other results.

GLOBAL EXISTENCE IN SOBOLEV SPACES FOR A CLASS OF NONLINEAR KIRCHHOFF EQUATIONS

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The nonlinear Kirchhoff equation $u_{tt} - m(\|\nabla u\|^2_{L^2})\Delta u = 0$ is studied for initial data $(u, u_t)_{t=0} = (u_0, u_1)$ in the Sobolev spaces $H^s(\mathbf{R}^n) \times H^{s-1}(\mathbf{R}^n)$ with $s \geq 2$ and for smooth perturbations $m(\rho)$ of the Pokhozhaev function $m_0(\rho) = (k_1\rho + k_0)^{-2}$ with k_0, $k_1 > 0$. Global existence is shown when $\|u_1\|_{L^2}$ is large and m is close to m_0 in a suitable metric. Moreover, the asymptotic behavior of solutions is found as $t \to \pm\infty$. It turns out that the norms $\|\nabla u\|_{L^2}$ grow like $|t|$, so the propagation speeds decrease like t^{-2} and the waves remain trapped in bounded regions.

This is a joint work with Lubin Vulkov from Rousse State University.