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Diffusion phenomena for the wave equation with structural damping in the $L^p - L^q$ framework

Abstract

The goal of this talk is to explain the *diffusion phenomena* for the wave equation with structural damping

$$u_{tt} - \triangle u + 2a(-\triangle)^\sigma u_t = 0\,, \qquad u(0,x) = u_0(x)\,, \quad u_t(0,x) = u_1(x)\,, \tag{1}$$

with $a > 0$ and $\sigma \in (0, 1/2)$. We show that u has a heat-type profile for low frequencies, i.e., u behaves like the solution v to

$$v_t + \frac{1}{2a}(-\triangle)^{1-\sigma} v = 0\,, \qquad v(0,x) = v_0(x)\,,$$

for suitable choice of initial data v_0. More precisely, we derive $L^p - L^q$ decay estimates for the difference $u - v$ and its time and space derivatives, where $1 \leq p \leq q \leq \infty$, possibly not on the conjugate line, satisfying some additional condition related to σ. In particular, we show that, under suitable assumptions on p, q, σ, a *double diffusion phenomenon* appears, that is, the difference $u - v$ behaves like the solution to

$$w_t + 2a\,(-\triangle)^\sigma w = 0\,, \qquad w(0,x) = w_0(x)\,,$$

for a suitable choice of initial data w_0.

The motivation for this work was the results obtained in the articles [2], [3] and by a remark done by the first author of the present work [1] about the $L^2 - L^2$ decay estimates for the solution of (1). In [3] the authors got some $L^p - L^q$ estimates for the solution u of (1), with $1 \leq p \leq q \leq \infty$ and $\sigma \in (0, 1]$. The limit case $\sigma = 0$ in (1) corresponds to the classical damped wave, for which the *diffusion phenomena* was already obtained(see [2] and the references therein).

Bibliography

[1] M. D'Abbicco and M. R. Ebert, Diffusion phenomena for the wave equation with structural damping in the $L^p - L^q$ framework, preprint.

[2] T. Narazaki, *$L^p - L^q$ estimates for damped wave equations and their applications to semilinear problem*, J. Math. Soc. Japan 56, 586–626(2004).

[3] T. Narazaki and M. Reissig, *L^1 estimates for oscillating integrals related to structural damped wave models*, 41pp., accepted for publication in a Birkhäuser volume of invited papers.