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Initial-boundary value problems for Zakharov–Kuznetsov equation on the plane

Abstract

Initial-boundary value problems for Zakharov–Kuznetsov equation

$$u_t + u_{xxx} + u_{xyy} + uu_x = 0 \tag{1}$$

in various domains are considered and problems of well-posedness are studied. The most recent results are related to the problem in a layer $\Pi_T = (0,T) \times \Sigma$, where $\Sigma = \{(x,y) : x \in \mathbb{R}, 0 < y < L\}$ is a horizontal strip of a given width L and $T > 0$ – arbitrary. Initial condition

$$u(0,x,y) = u_0(x,y), \qquad (x,y) \in \Sigma, \tag{2}$$

and Dirichlet boundary condition

$$u(t,x,0) = u(t,x,L) = 0, \qquad (t,x) \in (0,T) \times \mathbb{R}, \tag{3}$$

are set.

For any $\alpha \geq 0$ introduce function spaces

$$L_2^\alpha = H^{0,\alpha} = \{\varphi \in L_2(\Sigma) : (1+x_+)^\alpha \varphi \in L_2(\Sigma)\},$$
$$H^{1,\alpha} = \{\varphi \in H^1(\Sigma) : \varphi, \varphi_x, \varphi_y \in L_2^\alpha\}$$

with natural norms (here $x_+ = \max(x,0)$). Solutions to the considered problem are constructed in spaces $X^{k,\alpha}(\Pi_T)$, $k = 0$ or 1, consisting of functions $u(t,x,y)$ such that

$$u \in C_w([0,T]; H^{k,\alpha}), \qquad \sup_{x_0 \in \mathbb{R}} \int_0^T \int_{x_0}^{x_0+1} \int_0^L |D^{k+1}u|^2 \, dydxdt < \infty$$

and if $\alpha > 0$ then, in addition,

$$(1+x)^{\alpha-1/2} |D^{k+1}u| \in L_2\big((0,T) \times (0,+\infty) \times (0,L)\big),$$

where $|D^k\varphi| = \Big(\sum\limits_{k_1+k_2=k} (\partial_x^{k_1}\partial_y^{k_2}\varphi)^2\Big)^{1/2}$.

Theorem 1. *Let $u_0 \in L_2^\alpha$ for a certain $\alpha \geq 0$. Then there exists a weak solution to problem (1)–(3) in the space $X^{0,\alpha}(\Pi_T)$.*

Theorem 2. *Let $u_0 \in H^{1,\alpha}$ for a certain $\alpha \geq 0$ and $u_0|_{y=0} = u_0|_{y=L} = 0$. Let $u_0 \in L_2^\alpha$ for a certain $\alpha \geq 0$. Then there exists a weak solution to problem (1)–(3) in the space $X^{1,\alpha}(\Pi_T)$. If $\alpha \geq 1/2$ such a solution is unique.*

Similar rusults are established for other types of boundary conditions (Neumann and perodic).