

■ **First author** Fumihiko Hirosawa - Yamaguchi University, Japan, email: hirosawa@yamaguchi-u.ac.jp,
Second author Haruhisa Ishida - The University of Electro-Communications, Japan, e-mail: ishida@uec.ac.jp

On second order weakly hyperbolic equations and the ultradifferentiable classes

Abstract

Let us consider the Cauchy problem for second order weakly hyperbolic equation with time dependent coefficient:

$$\begin{cases} (\partial_t^2 - a(t)^2 \Delta) u = 0, & (t, x) \in (0, T] \times \mathbf{R}^n, \\ (u(0, x), u_t(0, x)) = (u_0(x), u_1(x)), & x \in \mathbf{R}^n, \end{cases} \quad (1)$$

where $a(t) \in C^\infty([0, T])$ satisfies $C^{-1}\lambda(t) \leq a(t) \leq C\lambda(t)$ with a positive constant $C > 1$ and a strictly decreasing function $\lambda(t)$ satisfying $\lambda(T) = 0$. Moreover, we assume that

$$|a^{(k)}(t)| \leq \lambda(t) M_k \rho(t)^k \quad (k = 1, 2, \dots)$$

for a positive function $\rho(t)$ and a logarithmical convex sequence $\{M_k\}$. Our main purpose is to describe the weight function μ for the estimate

$$|(\hat{u}(t, \xi), \hat{u}_t(t, \xi))| \leq e^{\mu(\langle \xi \rangle)} |(\hat{u}_0(\xi), \hat{u}_1(\xi))|,$$

which implies the well-posedness of (1) in μ -ultradifferentiable class of Beurling-Roumieu type, by $\lambda(t)$, $\rho(t)$ and $\{M_k\}$.

BIBLIOGRAPHY

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