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Recent results on uniqueness and continuous dependence in the Cauchy problem for backward-parabolic operators with low-regular coefficients

Abstract

We consider the Cauchy problem for backward-parabolic operators

$$\mathcal{P}u = \partial_t u + \sum_{i,j=1}^{n} \partial_{x_i}(a_{i,j}(t,x)\partial_{x_j} u) + \sum_{k=1}^{n} b_k(t,x)\partial_k u + c(t,x)u$$

and look for sufficient and necessary conditions to ensure the uniqueness of the solutions or the continuous dependence [5] of the solutions on the Cauchy data. We are especially interested in the connections between the regularity of the principal part coefficients and the mentioned properties. It is almost classical that the questions about uniqueness and stability have a positive answer if $a_{i,j}(t,x)$ are Lipschitz continuous with respect to time and bounded with respect to the spatial variable [1]. We follow two possibilities to weaken the Lipschitz condition:

(P1) *Local irregularity*: Suppose that $a_{ij} = a_{ij}(t)$ with $|F(t)d_t a_{ij}(t)| \leq C_{small}$, where F is a suitable function like $F(t) = t$ or $F(t) = t^2$.

(P2) *Global irregularity*: Suppose that $a_{ij} = a_{ij}(t,x) \in C^{\mu}([0,T], L^{\infty}(\mathbb{R}^n) \cap L^{\infty}([0,T], C^{\omega}(\mathbb{R}^n))$, where we investigate also the possible interactions between ω and μ.

We will illustrate the necessity of our conditions by suitable counterexamples. To prove our results we will use the Carleman estimate method and to prove suitable Carleman estimates we will use, e.g, Bony's para-differential calculus. the results about local irregularity are connected to singular Carleman weight functions. We will conclude the talk with some recent result obtained with K. Yagdjian on uniqueness in the Cauchy problem for degenerate elliptic operators generalizing a zone-method approach developed in [6].

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