■ **Mitsuru Sugimoto** Nagoya University, Nagoya, Japan, email: sugimoto@math.nagoya-u.ac.jp,

A vector fields approach to smoothing and decaying estimates for equations in anisotropic media

Abstract

It is well known that the vector fields

$$\Omega = x \wedge D = (\Omega_{ij})_{i<j}\,, \qquad \Omega_{ij} = x_i D_j - x_j D_i$$

commute with the Laplacian $-\Delta$. Hence we have

$$Pu = f \quad \Rightarrow \quad P(\Omega u) = \Omega f,$$

where $P = p(D_t, -\triangle)$, and in this way we can control the growth/decaying order of solution u to the equation $Pu = f$. This fact was actually used to induce some decaying estimates for the wave equation and smoothing estimates for the Scrödinger equation. In this talk, we will discuss how to trace this idea for equations with the Laplacian $-\Delta$ replaced by general elliptic (pseudo-)differential operators. Such situation naturally arises in the equation of linear elasticity for crystals and Maxwell equations in anisotropic media. In general, elliptic operators do not always have corresponding vector fields which commute with them, but some useful lemma and its application will be stated.