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Degenerate pseudodifferential operators of Vishik-Grushin type

Abstract

We develop a symbol calculus for a class of pseudodifferential operators that degenerate in a specific way along a regular submanifold. Differential operators L in this class are of the form

$$L = \sum_{|\alpha|+|\beta| \leq m} \sum_{|\gamma| \geq |\alpha|+|\beta|(l_*+1)-p} a_{\alpha\beta\gamma}(x,y)\, x^\gamma D_x^\alpha D_y^\beta,$$

in local coordinates $(x,y) \in \mathbb{R}^d \times \mathbb{R}^q$ near $x=0$, where $a_{\alpha\beta\gamma} \in C^\infty(\mathbb{R}^d \times \mathbb{R}^q)$ and the degeneracy occurs at $x=0$. Here, $l_* \in \mathbb{Q}_+$ describes the kind of degeneracy under study, $m \in \mathbb{N}_0$, $p \in \mathbb{Z}$, and $p \leq m$. For instance, one has $d=1$, $l_* = 1/2$, $m=p=2$ for the Tricomi operator $\partial_x^2 + x\Delta_y$.

As an application, well-posedness of a certain class of boundary-value problems for PDEs of mixed type, where the hyperbolic region is sandwiched between elliptic regions, is proved.

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