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ORGANIZERS: MARCELLO D'ABBICCO (BARI, SÃO PAULO), FUMIHIKO HIROSAWA (YAMAGUCHI), MICHAEL REISSIG (FREIBERG)

Short description of the Session

The goal of the session is to discuss the state-of-the-art of qualitative properties of solutions of linear and non-linear evolution models. We have in mind results for dispersive equations, σ-evolution equations and parabolic equations as well. Among other things well-posedness, asymptotic profile, blow-up behavior and the influence of low regular coefficients are of interest.

Plenary talks

Interpolation of Morrey and Related Smoothness Spaces

Winfried Sickel
Institute of Mathematics, Friedrich Schiller University Jena, Ernst-Abbe-Platz 2, D-07743 Jena, Germany
email: `winfried.sickel@uni-jena.de`

Abstract We study the interpolation of Morrey spaces and some smoothness spaces based on Morrey spaces, e. g., Besov-type and Triebel-Lizorkin-type spaces. Various interpolation methods, including the real and complex method, the $\pm$-method and the Peetre-Gagliardo method, are studied in such a framework. Special emphasize is given to the quasi-Banach case and to the interpolation property. This is joint work with Dachun Yang and Wen Yuan from Beijing Normal University.

Integral transform approach to the time-dependent partial differential equations

Karen Yagdjian
University of Texas-Pan American, Edinburg, U.S.A.
email: `yagdjian@utpa.edu`

Abstract In this talk we present an integral transform that allows to write solutions of the partial differential equation with variable coefficients via solutions of simpler equation.

Consider for the smooth function $f = f(x,t)$ the solution $w = w(x,t;b)$ to the problem

(1) $$w_{tt} - A(x,\partial_x)w = 0,\ \ w(x,0;b) = f(x,b),\ \ w_t(x,0;b) = 0,\ \ t \in [0,T_1],\ \ x \in \Omega \subseteq \mathbb{R}^n,$$

with the parameter $b \in [t_{in}, T]$, $0 \le t_{in} < T \le \infty$, and with $0 < T_1 \le \infty$. Here Ω is a domain in $\mathbb{R}^n$, while $A(x, \partial_x) = \sum_{|\alpha| \le m} a_\alpha(x) \partial_x^\alpha$ is the partial differential operator with smooth coefficients, $a_\alpha \in C^\infty(\Omega)$. We are going to present the integral operator

$$\mathcal{K}[w](x,t) = \int_{t_{in}}^{t} db \int_0^{|\phi(t)-\phi(b)|} K(t;r,b) w(x,r;b)\, dr, \quad x \in \Omega,\ t \in [t_{in}, T], \tag{2}$$

which maps the function $w = w(x, r; b)$ into the solution $u = u(x,t)$ of the equation

$$u_{tt} - a^2(t) A(x, \partial_x) u + M^2 u = f, \qquad x \in \Omega\,,\ t \in [t_{in}, T]\,,$$

where $a^2(t) = e^{\pm 2t}$, t^ℓ, $\ell \in \mathbb{C}$, $M \in \mathbb{C}$. In fact, the function $u = u(x,t)$ takes initial values

$$u(x, t_{in}) = 0,\ u_t(x, t_{in}) = 0, \quad x \in \Omega\,.$$

In (2), $\phi = \phi(t)$ is a distance function produced by $a = a(t)$, that is $\phi(t) = \int_{t_{in}}^t a(\tau)\, d\tau$. Moreover, we also introduce the corresponding operators, which generate solutions of the source-free equation and takes non-vanishing initial values.

We illustrate this approach by application to several model equations. In particular, we give applications to the generalized Tricomi equation, the Klein-Gordon and wave equations in the curved spacetimes such as de Sitter, Einstein-de Sitter, anti-de Sitter, Schwarzschild, and Schwarzschild-de Sitter spacetimes. The particular version of this transform was used in a series of papers [1, 2, 3, 4, 5] to investigate in a unified way several linear and semilinear equations. The results on the global existence of the small data solutions of the Cauchy problem for the semilinear Tricomi equation, the system of semilinear Klein-Gordon equations in the de Sitter spacetime, were established. The relations to the Higuchi bound and Huygens ‘ Principle were revealed as well.

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Invited talks

Strichartz estimates for the Germain-Lagrange equation with nonlinear memory

Marcello D'Abbicco
Universidade de São Paulo (USP), Ribeirão Preto - SP - Brasil
email: m.dabbicco@gmail.com

Abstract We apply linear Strichartz estimates [1] to study the local existence of the solution to

$$\begin{cases} u_{tt} + \Delta^2 u = c_\gamma \int_0^t (t-s)^{-\gamma} |u(s,x)|^p \, ds \quad t \geq 0, \; x \in \mathbb{R}^n, \\ u(0,x) = u_0(x), \\ u_t(0,x) = u_1(x), \end{cases}$$

where $\gamma \in (0,1)$ and $c_\gamma > 0$. The right-hand side represents a nonlinear memory term, in particular a fractional integral of $|u|^p$ for a suitable choice of c_γ. For $n \geq 5$, $s \in [0,2]$, and $\gamma \in (1/2, 1)$, we find the local existence of the H^s solution in the sub-critical range

$$1 + \frac{4}{n} \leq p < 1 + \frac{4(3-\gamma)}{n-2s},$$

provided that $u_0 \in H^s$ and $u_1 \in \dot{H}^{s-2} \cap \dot{H}^{-2}$. At the limit $\gamma \to 1$, we may replace the right-hand side by $|u(t,x)|^p$. In this latter case, we may also prove the global existence of the solution into $\dot{H}^s$ in the critical case $p = 1 + 8/(n-2s)$.

This is a joint paper with Sandra Lucente from University of Bari.

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Semi-Linear Systems of Weakly Coupled Classical Waves

Mohammed Djaouti, Abdelhamid
Technical University Bergakademie Freiberg, Freiberg, Germany
email: djaouti_abdelhamid@yahoo.fr

Abstract

Let us consider the following model of semi-linear systems of weakly coupled classical waves

$$\begin{cases} u_{tt} - \Delta u + b_1(t) u_t = |v|^p, (t,x) \in [0,\infty), x \in \mathbb{R}^n \\ v_{tt} - \Delta v + b_2(t) v_t = |u|^q \\ (u, u_t, v, v_t)(0,x) = (u_0, u_1, v_0, v_1)(x), x \in \mathbb{R}^n \end{cases}$$

for $n \leq 4$ and $p, q > 1$. Our aim is to investigate the global existence of $\mathcal{C}([0,\infty), H^1) \cap \mathcal{C}^1([0,\infty), L^2)$ solutions for small initial data. In particular, we suppose an effective dissipation (i.e. $b_1(t), b_2(t)$ are positive, monotone and $|b_1'(t)| = o(b_1^2(t)), |b_2'(t)| = o(b_2^2(t))$ as $t \mapsto \infty$) for both damping terms of the system.

We use Matsumura type estimates for a family of parameter-dependent Cauchy problems along the lines of [1] to estimate the decay of the solutions under some conditions on p, q depending on the classical Fujita exponent $p_{Fuj}(n)$ or $q_{Fuj}(n)$.

We show further that for $p \neq q$, $\min(p,q) < p_{Fuj}(n) < \max(p,q)$ one obtains a loss of decay of the solutions compared to the case $p = q$, $p > q_{Fuj}(n)$ and the scalar case.

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The external damping problems with general powers of the Laplacian

Pham Trieu Duong
Hanoi National University of Education, Hanoi, Vietnam
email: duongptmath@hnue.edu.vn

Abstract We present in this talk the survey on the following external damping problem

$$\begin{aligned} u_{tt} + (-\Delta)^\sigma u + u_t = ||D|^a u|^p \\ u(0,x) = u_0(x) \\ u_t(0,x) = u_1(x) \end{aligned}$$

with the assumption on the powers: $0 < a < \sigma,\ p > 1$, as previously, but the parameter σ belongs new range $\sigma \in (1, \infty)$. In the literatures that are devoted to the similar model with diffusion properties, the range $0 < \sigma < 1$ is stated frequently as a default, since only inside this restricted range, the operator $(-\Delta)^\sigma$ is *stable*. However other extended values of parameter σ occur too in the real-world problems and it gives the interests in study. We present our method of research instead of applying the statistics and probability notions, as it was proved successfully for the classical case $\sigma \in (0, 1)$. Our results indicate besides which essential conditions for differential operators that would influence on the decay estimates and the solvability as well for these initial value problems.

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On Zakharov–Kuznetsov equation on the plane and in the space

Andrei Faminskii
Peoples' Friendship University of Russia, Moscow, Russia
email: afaminskii@sci.pfu.edu.ru

Abstract Zakahrov–Kuznetsov (ZK) equation is one of the variants of multi-dimensional generalizations of Korteweg–de Vries equation. We consider the cases of two spatial dimensions (2D-ZK)

$$u_t + u_{xxx} + u_{xyy} + uu_x = 0$$

and three spatial dimensions (3D-ZK)

$$u_t + u_{xxx} + u_{xyy} + u_{xzz} + uu_x = 0.$$

These equations describe nonlinear waves in dispersive media, propagating in x-direction with deformations in transversal directions.

The theory of initial-boundary value problems is much better developed for 2D-ZK, although it is also far from being complete. The most complicated situation here is for domains, where the variable y is considered on a bounded interval. Certain results on global-well-posedness of such problems were obtained, for example, in [1].

Recently similar results were established for 3D-ZK. Let $\Omega \subset \mathbb{R}^2_{y,z}$ be a bounded domain with a sufficiently smooth boundary, $\Sigma = \mathbb{R}_x \times \Omega$. Consider an initial-boundary value problem with initial and boundary conditions

$$u\big|_{t=0} = u_0(x,y,z), \qquad u\big|_{(0,T)\times\partial\Sigma} = 0$$

for an arbitrary $T > 0$.

Theorem 1. *Let $(1+x_+)^{3/4}u_0 \in H^1_0(\Sigma)$. Then there exists a unique weak solution to this problem for 3D-ZK such that $(1+x_+)^{3/4}u \in L_\infty(0,T;H^1_0(\Sigma))$. Moreover, $u \in L_2(0,T;H^2(I\times\Omega))$ for any bounded interval $I \subset \mathbb{R}$.*

Similar result holds for the initial value problem.

Properties of large-time decay of solutions to 2D-ZK and 3D-ZK are also considered.

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Wave equation with time dependent propagation speed asymptotically monotone functions

Laila Fitriana
Yamaguchi University, Yamaguchi, Japan
email: lailaherman@yahoo.com.

Abstract:
We consider the estimates for the wave equation with time dependent propagation speed, if the propagation speed is a constant, then the energy conservation $E(t) = E(0)$ holds. However, the energy conservation does not hold in general for variable propagation speeds. The purpose of this talk is to apply the propagation speed behaves asymptotically as a monotone decreasing function. The main goal is to extend the stabilization properties in paper [2].

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Semilinear Hyperbolic Equations in De Sitter Spacetime

Anahit Galstyan
University of Texas-Pan American, 1201 W. University Drive, Edinburg, TX 78539, USA
email: agalstyan@utpa.edu

Abstract We present some results on the semilinear massless waves propagating in the de Sitter spacetime. The global in time existence of the solutions for the Klein-Gordon equation in the de Sitter spacetime is known (due to Yagdjian, Nakamura, Galstian) with very weak restriction on the order of nonlinearity. However, the existence of such solutions for the Cauchy problem for the semilinear massless equation is still an open problem. We give the estimate for the lifespan of the solutions if the exponent of nonlinearity is less than the critical value given by Strauss conjecture. The case of the hyperbolic spatial part of the manifold is especially interesting since it appears in some cosmological models. We present the lifespan estimate in this case as well.

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Hyperbolic equations and systems with non-regular coefficients

Claudia Garetto
Loughborough University, Loughborough, UK
email: c.garetto@lboro.ac.uk

Abstract This talk is a survey on some recent well-posedness results for weakly linear hyperbolic equations and systems with non-regular (less than Hölder) time dependent coefficients. First we focus on second order equations with distributional coefficients ([1] in collaboration with Michael Ruzhansky) then we pass to consider higher order equations and first order systems with bounded roots/eigenvalues [2]. A notion of very weak solution is introduced which is consistent with the classical Gevrey or ultradistributional solution whenever it exists.

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Decay rates for solutions of semilinear parabolic equations

Marina Ghisi
University of Pisa, Pisa, Italy
email: ghisi@dm.unipi.it

Abstract We consider an abstract first order evolution equation in a Hilbert space in which the linear part is represented by a self-adjoint nonnegative operator A with discrete spectrum, and the nonlinear

term has order greater than one at the origin. We investigate the asymptotic behavior of solutions. We prove that two different regimes coexist. Close to the kernel of A the dynamic is governed by the nonlinear term, and solutions (when they decay to 0) decay as negative powers of t. Close to the range of A, the nonlinear term is negligible, and solutions behave as solutions of the linearized problem. This means that they decay exponentially to 0, with a rate and an asymptotic profile given by a simple mode, namely a one-frequency solution of the linearized equation ([1]).

The abstract results apply to semilinear parabolic equations, as for example:

$$u_t - \triangle u + |u|^p u = 0$$

with homogeneous Neumann boundary conditions and

$$u_t - \triangle u - \lambda_1 u + |u|^p u = 0$$

with homogeneous Dirichlet boundary conditions, where λ_1 denotes the first eigenvalue of (minus) the Dirichlet Laplacian.

In such concrete cases we also describe the asymptotic profile of slow solutions, and we show that the set of initial data giving rise to fast solutions is a graph of codimension one in the space of all continuous initial data ([2]).

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Effectiveness of time-dependent damping terms for second order evolution equations

Massimo, Gobbino
University of Pisa, Pisa, Italy
email: massimo.gobbino@unipi.it

Abstract We consider abstract evolution equations of the form

$$u''(t) + 2\delta(t)u'(t) + Au(t) = 0$$

where H is a Hilbert space, A is a self-adjoint linear operator on H with dense domain, and $\delta : [0, +\infty) \to [0, +\infty)$ is a measurable function. We always assume that the spectrum of A is a finite set, or an unbounded increasing sequence of positive eigenvalues.

Our aim is designing the coefficient $\delta(t)$ in such a way that *all* solutions decay as fast as possible as $t \to +\infty$. We discover that constant coefficients do not achieve the goal, as well as time-dependent coefficients that are too big if compared with the smallest eigenvalue of A. Indeed the fastest decay rate that can be achieved through coefficients of this type is $te^{-\nu t}$, where ν^2 is the smallest eigenvalue of A.

On the contrary, pulsating coefficients which alternate big and small values in a suitable way prove to be more effective, in the sense that they can force all solutions to decay faster than all exponentials.

Our theory applies to ordinary differential equations, systems of ordinary differential equations, and partial differential equations of hyperbolic type.

(This is joint work with Marina Ghisi (Pisa) and Alain Haraux (Paris 6))

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On the energy estimates for the wave equations with decaying propagation speed

Fumihiko Hirosawa
Yamaguchi University, Yamaguchi, Japan
email: hirosawa@yamaguchi-u.ac.jp

Abstract:
We consider the energy estimates for the wave equation with time dependent propagation speed. It is known that the asymptotic behavior of the energy is determined by the interactions of the properties of the propagation speed: smoothness, oscillation and the difference from an auxiliary function. The main purpose of the article is that if the propagation speed behaves asymptotically as a monotone decreasing function, then we can extend the preceding results to allow faster oscillating coefficients. Moreover, we prove that the regularity of the initial data in the Gevrey class can essentially contribute for the energy estimate.

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Conditional stability for backward-parabolic equations

Christian Jäh
Technical University Bergakademie Freiberg, Freiberg, Germany
email: christian.jaeh@math.tu-freiberg.de

Abstract
We consider the Cauchy problem for backward-parabolic operators

$$\begin{cases} \mathcal{P}u = \partial_t u + \sum\limits_{i,j=1}^{n} \partial_{x_i}(a_{ij}(t,x)\partial_{x_j} u) + \sum\limits_{k=1}^{n} b_k(t,x)\partial_k u + c(t,x)u \\ u(0,x) = u_0(x), \end{cases}$$

with

$$\sum_{j,i=1}^{n} a_{ij}(t,x)\xi_i\xi_j \geq \lambda|\xi|^2, \quad \forall (t,x,\xi) \in [0,T] \times \mathbb{R}^n_x \times \mathbb{R}^n_\xi, \quad \lambda \in (0,1]$$

and look for sufficient and (almost) necessary conditions to ensure conditional continuous dependence of solutions

$$u \in C^0([0,T], L^2(\mathbb{R}^n_x)) \cap C^0([0,T), H^1(\mathbb{R}^n_x)) \cap C^1([0,T), L^2(\mathbb{R}^n_x))$$

on the Cauchy datum $u_0 \in L^2(\mathbb{R}^n_x)$. Specifically, we will prove a conditional stability result in the sense of John.

We are especially interested in the connections between the regularity of the principal part coefficients and the conditional stability. It is almost classical that conditional stability (Hölder stability) holds if $a_{ij}(t,x)$ are Lipschitz continuous with respect to the time and sufficiently regular with respect to the spatial variables.

In this paper [2], we study the possible stability result if we weaken the the Lipschitz condition with respect to the time variable to an Osgood modulus of continuity μ, i.e.

$$\int_0^1 \frac{ds}{\mu(s)} = +\infty$$

and with respect to the spatial variables to Lipschitz continuity. This result generalizes [1, 3]. To prove our result, we will prove suitable weighted energy estimates. Do deal with the low regularity of the principal part coefficients, we apply a suitable form of Bony's paradifferential calculus.

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On the Euler–Poisson equations

Lavi Karp
Department of Mathematics, ORT Braude College, P.O. Box 78, 2161002 Karmiel, Israel
email: `karp@braude.ac.il`

Abstract The Euler–Poisson equations

$$\begin{cases} \rho_t + \nabla \cdot (\rho u) = 0 \\ (\rho u)_t + \nabla \cdot (\rho u \otimes u) + \nabla P(\rho) = -\rho \nabla \Phi \\ \Delta \Phi = 4\pi\rho \end{cases}$$

is a hyperbolic–elliptic system that describes the dynamic behavior of many important physical flows, and in particular the evolution of a star regarded as an ideal gas with self gravitation. It is known that this can be written a symmetric hyperbolic system and therefore has a well–posed initial problem provided that the density is bounded below by a positive constant. However, in astrophysical context the density is expected to have compact support, or tend to zero at spatial infinity. It turns out that the system becomes degenerated whenever the density approaches zero. The talk will discuss a certain method to over come this difficulty and will present well-posedness for densities that decay to zero at infinity and have a finite mass.

This is a joint work with U. Brauer.

Nonlinear evolution equations and application to mathematical models

Akisato, Kubo
School of Health Sciences, Fujita Health University
email: `akikubo@fujita-hu.ac.jp`

Abstract In this talk we consider initial-Neumann boundary value problem of nonlinear evolution equations with strong dissipation and proliferation arising from mathematical biology and medicine formulated as

$$
(NE)\begin{cases}
u_{tt} = D\Delta u_t + \nabla\cdot(\chi(u_t, e^{-u})e^{-u}\nabla u) + \mu_1 u_t(1-u_t) & \\
 & in \ \ (x,t)\in \mathbf{\Omega}\times(0,\infty) \quad (1.1)\\
\dfrac{\partial}{\partial\nu}u|_{\partial\mathbf{\Omega}} = 0 & on \ \ \partial\mathbf{\Omega}\times(0,\infty) \quad (1.2)\\
u(x,0) = u_0(x), u_t(x,0) = u_1(x) & in \ \ \mathbf{\Omega} \quad (1.3)
\end{cases}
$$

where constants D, μ_1 are positive, $\mathbf{\Omega}$ is a bounded domain in R^n with a smooth boundary $\partial\mathbf{\Omega}$ and ν is the outer unit normal vector. Under appropriate regularity and boundedness conditions of the coefficient $\chi(u_t, e^{-u})$ of (1.1), we derive the energy estimate of (NE) and it enables us to show the global existence in time and asymptotic behavior of the solution. In the case of $\mu_1 = 0$ we studied the problem for a more restricted condition of $\chi(u_t, e^{-u})$ in [1]. In [2] our main concern is to seek the solution of the form of $u = t + v$. Our goal of this talk to consider other type of solutions to (NE), particularly considering into the logistic solution, or to study a more general case of (1.1). Finally we apply our result to mathematical models arising from biomedicine and discuss the behavior of cell migration.

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Models of thermo-diffusion in 1d

Yan Liu
Guangdong University of Finance, Guangzhou, China
email: ly801221@163.com

Abstract The goal of the paper is to study the Cauchy problem for 1d models of thermo-diffusion. We explain qualitative properties of solutions. In particular, we show which part of the model has a dominant influence on well-posedness, propagation of singularities, $L^p - L^q$ decay estimates on the conjugate line and on the diffusion phenomenon.

Multiple solutions for phase transition problems in higher dimensions

Xiaojun Lu[1] and David Yang Gao[2]
1. School of Economics and Management & Department of Mathematics, Southeast University, 211189, Nanjing, China
2. School of Science, Information Technology and Engineering
Federation University Australia, Ballarat, VIC 3353, Australia
email: lvxiaojun1119@hotmail.de

Abstract In this paper, we address the non-convex variational BVPs(boundary value problems) in higher dimensional Lipschitz domains $\Omega \subset \mathbb{R}^n$. For $n = 1$, lots of literature was devoted to the study of nonlinear elasticity of the Ericksen bar. Generally speaking, for $n \geq 2$, the strain-energy function is a non-convex non-homogeneous fourth-order polynomial when we consider the phase transitions of the mechanical models. Based on the newly developed methodology of canonical dual transformation, the non-convex variational problem has been converted into an algebraic problem, which can be solved completely. As a matter of fact, the uniqueness of the solution of the nonlinear elliptic equation does not hold since the divergence equation has many solutions. According to the dual curve for the algebraic equation,

a multi-solution criterion and the corresponding analytical solutions will be discussed in detail. It is worth noticing the identification and characterization of the global energy minimizer and the local energy extrema due to triality theory. As applications, we shall show several typical mechanical models with specific forcing terms and boundary conditions in 2-D domains by using numerical methods.

The Cauchy problem for nonlinear complex Ginzburg-Landau equations

Makoto Nakamura
Department of Mathematical sciences, Yamagata University, Yamagata, JAPAN
email: nakamura@sci.kj.yamagata-u.ac.jp

Abstract The Cauchy problem of nonlinear complex Ginzburg-Landau equations is considered in Sobolev spaces under the variance of the space-time. Some properties by the spatial expansion on energy solutions are remarked.

Uniform resolvent estimate for stationary dissipative wave equations in an exterior domain and their application to the principle of limiting amplitude

Hideo, Nakazawa
Nippon Medical School, Musashino, Japan
email: nakazawa-hideo@nms.ac.jp

Abstract Let Ω be an exterior domain of $\mathbb{R}^N$ ($N \geqq 2$) with star-shaped smooth boundary $\partial\Omega$. We consider the stationary problem of the form

$$\begin{cases} \left(-\Delta - i\kappa b(x) - \kappa^2\right) u(x) = f(x), & x \in \Omega, \\ u(x) = 0, & x \in \partial\Omega, \end{cases} \tag{3}$$

where $b(x)$ is a real-valued bounded smooth function on Ω and $\kappa \in \mathbb{C}$ denotes a spectral parameter. Moreover, it is assumed that the function $b(x)$ satisfies certain decay conditions.

The main result is the uniform resolvent estimate for (1). As an application of it, the principle of limiting amplitude follows. The above results are based on a joint work with Kiyoshi Mochizuki (Emeritus, Tokyo Metropolitan University).

Critical exponents for the Cauchy problem to weakly coupled system of wave equations with space or time dependent damping

Kenji, Nishihara
Waseda University, Tokyo, Japan
email: kenji@waseda.jp

Abstract In this talk we consider the Cauchy problem to the weakly coupled system of wave equations with space or time dependent damping

$$\begin{cases} u_{tt} - \Delta u + b(t,x)u_t = |v|^p, \\ v_{tt} - \Delta v + b(t,x)v_t = |u|^q, \quad (t,x) \in \mathbb{R}_+ \times \mathbb{R}^N, \\ (u, u_t, v, v_t)(0,x) = (u_0, u_1, v_0, v_1), \quad x \in \mathbb{R}^N, \end{cases}$$

where $p, q > 1$ and

$$\text{(I)} \;\; b(t,x) = \langle x \rangle^{-\alpha} \;\; (0 \le \alpha < 1) \quad \text{or} \quad \text{(II)} \;\; b(t,x) = (1+t)^{-\beta} \;\; (-1 < \beta < 1).$$

Our aim is to determine the critical exponents in those cases. Under the suitable conditions we will show that the critical exponents are given by

$$\Lambda := \max\left(\frac{p+1}{pq-1}, \frac{q+1}{pq-1}\right) = \frac{N-\alpha}{2} \ \text{ in (I)} \quad \text{or} \quad \Lambda = \frac{N}{2} \ \text{ in (II)}.$$

We note that the blow-up result is remained open and the critical exponents are not yet determined when $b(t,x) = \langle x\rangle^{-\alpha}(1+t)^{-\beta}$ $(0 < \alpha + \beta < 1, \alpha > 0, \beta > 0)$.

This talk is based on the joint work with Yuta Wakasugi.

Wave models with structural properties of the time-dependent potential

Nunes do Nascimento, Wanderley; Hirosawa, Fumihiko
TU Bergakademie Freiberg, Freiberg, Germany and Federal University of Sao Carlos, Sao Carlos, Brasil; Yamaguchi University, Yamaguchi, Japan
email: wnunesmg@yahoo.com.br

Abstract:
The aim of this talk is to apply C^m theory and stabilization condition for Klein-Gordon equation with non-effective time-dependent potential. The main goal is to generalize the paper [4] where the authors proved energy conservation dealing with "very slow oscillations" (according to the classification of Reissig and Yagdjian [2] and [3]) in the time-dependent potential. We are interested in the behavior of the energy as $t \to \infty$ for the coefficients bearing "very fast oscillations". Indeed, the energy has the same asymptotic behavior like in [4] considering very fast oscillations in the potential term under C^m properties and stabilization condition. Basically we perform a change of variable transforming the Klein-Gordon time-dependent problem into a damped wave time-depending problem and apply the technique are presented in the paper [1].

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Stabilization of the fourth order Schrödinger equation

B. Aksas and S.E. Rebiai
LTM, Department of Mathematics, Faculty of Sciences, University of Batna, 05000 Batna, Algeria
email: rebiai@hotmail.com

Abstract We study the uniform stabilization problem for the fourth order Schrödinger equation in a smooth bounded domain Ω of $\mathbb{R}^n$ with a suitable feedback control. This control is acting either on the boundary or on its neighborhood. For both cases, we show that the solutions decay exponentially in an appropriate energy space. The proof of these results combines multiplier techniques and compactness-uniqueness arguments.

Recent progress for semi-linear damped σ-evolution models

Michael Reissig
Technical University Bergakademie Freiberg, Freiberg, Germany
email: reissig@math.tu-freiberg.de

Abstract We discuss the Cauchy problem for semi-linear damped σ-evolution models

$$u_{tt} + (-\Delta)^\sigma u + b(t)(-\Delta)^\delta u_t = f(u, u_t, |D|^a u),\;\; u(0,x) = u_0(x),\;\; u_t(0,x) = u_1(x)$$

with different model power non-linearities, $a \in (0, \sigma]$. Our main issue is to determine the critical exponent dividing the range of admissible exponents into those producing, in general, a blow-up behavior of solutions and those allow the proof of global existence (in time) of small data solutions. Matsumura type estimates for solutions to parameter-dependent Cauchy problems are an important tool in our approach. We will explain how modern results of harmonic analysis can be used to treat the non-linear terms. Some discussion of optimality of our results and some open problems complete the talk.

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Unconditional global well-posedness for a nonlinear damped beam equation with decay property of the solution.

Hiroshi Takeda
Fukuoka Institute of Technology, Fukuoka, Japan
email: h-takeda@fit.ac.jp

Abstract In this talk, we consider a nonlinear beam equation with the weak damping term. Our aim is to show that the unconditional global well-posedeness for the small data and obtain the decay estimates of the solution in the same framework. Our method is based on the linear estimates in the weighted Sobolev space. In the linear estimates, we can observe not only the dissipative structure but also the regularity loss properties for the weighted estimates of the linearized solution. On the other hand, using the smoothing effects from the higher order term, we can prevent the extra regularity loss form the nonlinear interaction. We also show the large data blow up results, which implies that the smallness of the data is important for our situation.

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The lifespan of solutions to semilinear damped wave equations in one space dimension

Kyouhei Wakasa
Hokkaido University, Sapporo, Japan
email: wakasa@math.sci.hokudai.ac.jp

Abstract In this talk, we consider the initial value problem for semilinear damped wave equations in one space dimension. Ikeda & Wakasugi [1] and Wakasugi [2] have obtained an upper bound of the lifespan for the problem only in the subcritical case. The aim of this talk is to give an estimate of the upper bound of the lifespan in the critical case, and show the optimality of the upper bound. Also, we derive an estimate of the lower bound of the lifespan in the subcritical case which shows the optimality of the upper bound in [1] and [2]. Moreover, we show that the critical exponent changes when the initial data satisfies some symmetric assumption.

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A remark on the asymptotic profile of solutions to the damped wave equation with variable coefficients

Yuta Wakasugi
Nagoya University, Japan
email: yuta.wakasugi@math.nagoya-u.ac.jp

Abstract In this talk, we consider the Cauchy problem to the nonlinear damped wave equation

$$\begin{cases} u_{tt} - (a(x)u_x)_x + b(t)u_t = N(u, u_x, u_t), & t > 0, x \in \mathbb{R}, \\ (u, u_t)(0, x) = (u_0, u_1)(x), & x \in \mathbb{R} \end{cases}$$

and we study the asymptotic behavior of solutions as $t \to +\infty$. When $a(x) \equiv 1$, $b(t) = (1+t)^{-\beta}$ $(-1 < \beta < 1)$ and $N(u, u_x, u_t) = -|u|^{p-1}u$ $(p > 3)$, Nishihara [2] proved that the asymptotic profile of the solution with small initial data is given by the constant multiple of the heat kernel of the corresponding parabolic equation. In this talk, we extend the result of [2] to more general $a(x)$, $b(t)$ and $N(u, u_x, u_t)$. We show that if $a(x)$ has the positive limits $a_\pm$ as $x \to \pm\infty$, $b(t)$ behaves as $b(t) \sim (1+t)^{-\beta}$ with some $-1 < \beta < 1$, and the nonlinear term $N(u, u_x, u_t)$ can be regarded as a perturbation, then the asymptotic profile of the solution with small initial data is given by the constant multiple of the heat kernel of the corresponding parabolic equation. For the proof, following Gallay and Raugel [1], we use the scaling variables defined by

$$y = B(t)^{-1/2}x, \quad s = \log B(t) \quad \text{with } B(t) = \int_0^t \frac{d\tau}{b(\tau)} + 1$$

and we put $B(t)^{1/2}u(t,x) = v(s,y)$ and $b(t)B(t)^{3/2}u_t(t,x) = w(s,y)$. Then we have the first order system

$$\begin{cases} v_s - \dfrac{y}{2}v_y - \dfrac{1}{2}v = w, \\ \dfrac{e^{-s}}{b(t)^2}\left(w_s - \dfrac{y}{2}w_y - \dfrac{3}{2}w\right) - \dfrac{b'(t)}{b(t)^2}w + w = (a(e^{s/2}y)v_y)_y + \tilde{N}(t,v,w). \end{cases}$$

Then we show that the solution (v,w) can be expanded by eigenfunctions of the limiting operator

$$\tilde{\mathcal{L}}v = (\tilde{a}(y)v_y)_y + \frac{y}{2}v_y + \frac{1}{2}v,$$

where $\tilde{a}(y)$ is defined by $\tilde{a}(y) = a_-$ $(y<0)$ and $\tilde{a}(y) = a_+$ $(y>0)$.

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On t-dependent hyperbolic systems

Jens Wirth
University of Stuttgart, Stuttgart, Germany
email: jens.wirth@mathematik.uni-stuttgart.de

Abstract In the joint paper [1] with M. Ruzhansky we considered t-dependent hyperbolic systems and derived dispersive type estimates for time-dependent hyperbolic systems under rather generic assumptions for the full symbol of the operator within the hyperbolic zone. The assumptions concerning small frequencies (mild form of dissipativity) were rather artificial. In the light of the recent paper [2] with W. Nunes, we reformulate them in a more natural manner and consider the Cauchy problem

$$\mathrm{D}_t U = A(t,\mathrm{D}_x)U, \qquad U(0,\cdot) = U_0$$

for a vector valued function $U = U(t,x)$ having d components, $x \in \mathbb{R}^n$, $t \geq 0$. As usual $\mathrm{D} = -\mathrm{i}\partial$ denotes the Fourier derivative and conditions are imposed on the full symbol $A(t,\xi)$ of the Fourier multiplier $A(t,\mathrm{D}_x)$. There are two kinds of conditions involved,

- first: uniform strict hyperbolicity conditions on the large-frequency principal part accompanied by a technical condition implying a generalised energy conservation property for the hyperbolic zone;
- second: conditions involving a large-time, small frequency principal part

$$tA(t,\xi) \sim A_p, \qquad t \to \infty, t|\xi| \ll 1,$$

coming into play whenever the equation is close to be scale-invariant.

In combination, both conditions allow for an asymptotic construction of the fundamental solution of the problem and yield enough structural information to derive energy and dispersive type inequalities for its solutions.

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Singular limits in the Cauchy problem for the damped extensible beam equation

Shuji Yoshikawa
Ehime University, Ehime, Japan
email: yoshikawa@ehime-u.ac.jp

Abstract We discuss the Cauchy problem of the Ball model for an extensible beam ([1]):

$$\rho\partial_t^2 u + \partial_t u + \partial_x^4 u + \eta\partial_t\partial_x^4 u = \left(1 + \int_{\mathbb{R}} |\partial_x u|^2 dx + \eta \int_{\mathbb{R}} \partial_x u \partial_t \partial_x u dx\right)\partial_x^2 u, \qquad \text{in } \mathbb{R}^+ \times \mathbb{R}.$$

The aim is to investigate uniform singular limits as $\rho \to 0$ for the problem with the help of decay estimates. We will focus on introducing how to obtain the decay estimates for the limiting problem in $\rho = 0$ but $\eta > 0$. This talk is based on the results in [2] and [3].

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