Semi-Linear Systems of Weakly Coupled Classical Waves

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Abstract

Let us consider the following model of semi-linear systems of weakly coupled classical waves

$$\begin{cases} u_{tt} - \Delta u + b_1(t)u_t = |v|^p, (t,x) \in [0,\infty), x \in \mathbb{R}^n \\ v_{tt} - \Delta v + b_2(t)v_t = |u|^q \\ (u, u_t, v, v_t)(0,x) = (u_0, u_1, v_0, v_1)(x), x \in \mathbb{R}^n \end{cases}$$

for $n \leq 4$ and $p, q > 1$. Our aim is to investigate the global existence of $\mathcal{C}([0,\infty), H^1) \cap \mathcal{C}^1([0,\infty), L^2)$ solutions for small initial data. In particular, we suppose an effective dissipation (i.e. $b_1(t), b_2(t)$ are positive, monotone and $|b_1'(t)| = o(b_1^2(t)), |b_2'(t)| = o(b_2^2(t))$ as $t \mapsto \infty$) for both damping terms of the system.

We use Matsumura type estimates for a family of parameter-dependent Cauchy problems along the lines of [1] to estimate the decay of the solutions under some conditions on p, q depending on the classical Fujita exponent $p_{Fuj}(n)$ or $q_{Fuj}(n)$.

We show further that for $p \neq q$, $\min(p,q) < p_{Fuj}(n) < \max(p,q)$ one obtains a loss of decay of the solutions compared to the case $p = q$, $p > q_{Fuj}(n)$ and the scalar case.

Bibliography

[1] M. Dabbicco, S. Lucente, M. Reissig; *Semi-linear wave equations with effective dampling*, Chin. Ann. Math. 34B(3) (2013), 345–380.
[2] A. Matsumura; *On the asymptotic behavior of solutions of semi-linear wave equations*, Publ. RIMS. 12 (1976),169–189.
[3] K. Nishihara, Y. Wakasugi; *Critical exponant for the Cauchy problem to the weakly coupled wave system*, Nonlinear Analysis 108 (2014), 249–259.
[4] G. Todorova, B. Yordanov; *Critical Exponent for a Nonlinear Wave Equation with Damping*, Journal of Differ- ential Equations 174 (2001), 464489.
[5] J. Wirth; *Asymptotic properties of solutions to wave equations with time-dependent dissipation*, PhD Thesis, TU Bergakademie Freiberg, 2004.
[6] J. Wirth; *Wave equations with time-dependent dissipation II, Effective dissipation*, J. Differential Equations 232 (2007), 74–103.