On Zakharov–Kuznetsov equation on the plane and in the space

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Abstract Zakahrov–Kuznetsov (ZK) equation is one of the variants of multi-dimensional generalizations of Korteweg–de Vries equation. We consider the cases of two spatial dimensions (2D-ZK)

$$u_t + u_{xxx} + u_{xyy} + uu_x = 0$$

and three spatial dimensions (3D-ZK)

$$u_t + u_{xxx} + u_{xyy} + u_{xzz} + uu_x = 0.$$

These equations describe nonlinear waves in dispersive media, propagating in x-direction with deformations in transversal directions.

The theory of initial-boundary value problems is much better developed for 2D-ZK, although it is also far from being complete. The most complicated situation here is for domains, where the variable y is considered on a bounded interval. Certain results on global-well-posedness of such problems were obtained, for example, in [1].

Recently similar results were established for 3D-ZK. Let $\Omega \subset \mathbb{R}^2_{y,z}$ be a bounded domain with a sufficiently smooth boundary, $\Sigma = \mathbb{R}_x \times \Omega$. Consider an initial-boundary value problem with initial and boundary conditions

$$u\big|_{t=0} = u_0(x,y,z), \qquad u\big|_{(0,T)\times\partial\Sigma} = 0$$

for an arbitrary $T > 0$.

Theorem 1. *Let $(1+x_+)^{3/4}u_0 \in H^1_0(\Sigma)$. Then there exists a unique weak solution to this problem for 3D-ZK such that $(1+x_+)^{3/4}u \in L_\infty(0,T;H^1_0(\Sigma))$. Moreover, $u \in L_2(0,T;H^2(I\times\Omega))$ for any bounded interval $I \subset \mathbb{R}$.*

Similar result holds for the initial value problem.

Properties of large-time decay of solutions to 2D-ZK and 3D-ZK are also considered.

Bibliography

[1] Baykova, E.S. and Faminskii, A.V., *On initial-boundary-value problems in a strip for the generalized two-dimensional Zakharov–Kuznetsov equation*, Adv. Differential Equ., **18**, 663-686 (2013).