Decay rates for solutions of semilinear parabolic equations

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Abstract We consider an abstract first order evolution equation in a Hilbert space in which the linear part is represented by a self-adjoint nonnegative operator A with discrete spectrum, and the nonlinear term has order greater than one at the origin. We investigate the asymptotic behavior of solutions. We prove that two different regimes coexist. Close to the kernel of A the dynamic is governed by the nonlinear term, and solutions (when they decay to 0) decay as negative powers of t. Close to the range of A, the nonlinear term is negligible, and solutions behave as solutions of the linearized problem. This means that they decay exponentially to 0, with a rate and an asymptotic profile given by a simple mode, namely a one-frequency solution of the linearized equation ([1]).

The abstract results apply to semilinear parabolic equations, as for example:

$$u_t - \triangle u + |u|^p u = 0$$

with homogeneous Neumann boundary conditions and

$$u_t - \triangle u - \lambda_1 u + |u|^p u = 0$$

with homogeneous Dirichlet boundary conditions, where λ_1 denotes the first eigenvalue of (minus) the Dirichlet Laplacian.

In such concrete cases we also describe the asymptotic profile of slow solutions, and we show that the set of initial data giving rise to fast solutions is a graph of codimension one in the space of all continuous initial data ([2]).

Bibliography

[1] Ghisi M., Gobbino M., Haraux A., *A description of all possible decay rates for solutions of some semilinear parabolic equations.*, J. Math. Pures Appl., **103**, 864-899 (2015).

[2] Ghisi M., Gobbino M., Haraux A., *in preparation*.