

Effectiveness of time-dependent damping terms for second order evolution equations

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Abstract We consider abstract evolution equations of the form

$$u''(t) + 2\delta(t)u'(t) + Au(t) = 0$$

where H is a Hilbert space, A is a self-adjoint linear operator on H with dense domain, and $\delta : [0, +\infty) \rightarrow [0, +\infty)$ is a measurable function. We always assume that the spectrum of A is a finite set, or an unbounded increasing sequence of positive eigenvalues.

Our aim is designing the coefficient $\delta(t)$ in such a way that *all* solutions decay as fast as possible as $t \rightarrow +\infty$. We discover that constant coefficients do not achieve the goal, as well as time-dependent coefficients that are too big if compared with the smallest eigenvalue of A . Indeed the fastest decay rate that can be achieved through coefficients of this type is $te^{-\nu t}$, where ν^2 is the smallest eigenvalue of A .

On the contrary, pulsating coefficients which alternate big and small values in a suitable way prove to be more effective, in the sense that they can force all solutions to decay faster than all exponentials.

Our theory applies to ordinary differential equations, systems of ordinary differential equations, and partial differential equations of hyperbolic type.

(This is joint work with Marina Ghisi (Pisa) and Alain Haraux (Paris 6))

BIBLIOGRAPHY

- [1] M. Ghisi, M. Gobbino, A. Haraux. *The remarkable effectiveness of time-dependent damping terms for second order evolution equations*, in preparation.