Conditional stability for backward-parabolic equations

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Abstract

We consider the Cauchy problem for backward-parabolic operators

$$\begin{cases} \mathcal{P}u = \partial_t u + \sum_{i,j=1}^{n} \partial_{x_i}(a_{ij}(t,x)\partial_{x_j}u) + \sum_{k=1}^{n} b_k(t,x)\partial_k u + c(t,x)u \\ u(0,x) = u_0(x), \end{cases}$$

with

$$\sum_{j,i=1}^{n} a_{ij}(t,x)\xi_i\xi_j \geq \lambda|\xi|^2, \quad \forall (t,x,\xi) \in [0,T] \times \mathbb{R}^n_x \times \mathbb{R}^n_\xi, \quad \lambda \in (0,1]$$

and look for sufficient and (almost) necessary conditions to ensure conditional continuous dependence of solutions

$$u \in C^0([0,T], L^2(\mathbb{R}^n_x)) \cap C^0([0,T), H^1(\mathbb{R}^n_x)) \cap C^1([0,T), L^2(\mathbb{R}^n_x))$$

on the Cauchy datum $u_0 \in L^2(\mathbb{R}^n_x)$. Specifically, we will prove a conditional stability result in the sense of John.

We are especially interested in the connections between the regularity of the principal part coefficients and the conditional stability. It is almost classical that conditional stability (Hölder stability) holds if $a_{ij}(t,x)$ are Lipschitz continuous with respect to the time and sufficiently regular with respect to the spatial variables.

In this paper [2], we study the possible stability result if we weaken the the Lipschitz condition with respect to the time variable to an Osgood modulus of continuity μ, i.e.

$$\int_0^1 \frac{ds}{\mu(s)} = +\infty$$

and with respect to the spatial variables to Lipschitz continuity. This result generalizes [1, 3]. To prove our result, we will prove suitable weighted energy estimates. Do deal with the low regularity of the principal part coefficients, we apply a suitable form of Bony's paradifferential calculus.

Bibliography

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