Multiple solutions for phase transition problems in higher dimensions

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Abstract In this paper, we address the non-convex variational BVPs(boundary value problems) in higher dimensional Lipschitz domains $\Omega \subset \mathbb{R}^n$. For $n = 1$, lots of literature was devoted to the study of nonlinear elasticity of the Ericksen bar. Generally speaking, for $n \geq 2$, the strain-energy function is a non-convex non-homogeneous fourth-order polynomial when we consider the phase transitions of the mechanical models. Based on the newly developed methodology of canonical dual transformation, the non-convex variational problem has been converted into an algebraic problem, which can be solved completely. As a matter of fact, the uniqueness of the solution of the nonlinear elliptic equation does not hold since the divergence equation has many solutions. According to the dual curve for the algebraic equation, a multi-solution criterion and the corresponding analytical solutions will be discussed in detail. It is worth noticing the identification and characterization of the global energy minimizer and the local energy extrema due to triality theory. As applications, we shall show several typical mechanical models with specific forcing terms and boundary conditions in 2-D domains by using numerical methods.