Critical exponents for the Cauchy problem to weakly coupled system of wave equations with space or time dependent damping

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Abstract In this talk we consider the Cauchy problem to the weakly coupled system of wave equations with space or time dependent damping

$$\begin{cases} u_{tt} - \Delta u + b(t,x)u_t = |v|^p, \\ v_{tt} - \Delta v + b(t,x)v_t = |u|^q, \quad (t,x) \in \mathbb{R}_+ \times \mathbb{R}^N, \\ (u, u_t, v, v_t)(0,x) = (u_0, u_1, v_0, v_1), \quad x \in \mathbb{R}^N, \end{cases}$$

where $p, q > 1$ and

$$\text{(I)} \quad b(t,x) = \langle x \rangle^{-\alpha} \quad (0 \le \alpha < 1) \qquad \text{or} \qquad \text{(II)} \quad b(t,x) = (1+t)^{-\beta} \quad (-1 < \beta < 1).$$

Our aim is to determine the critical exponents in those cases. Under the suitable conditions we will show that the critical exponents are given by

$$\Lambda := \max\left(\frac{p+1}{pq-1}, \frac{q+1}{pq-1}\right) = \frac{N-\alpha}{2} \ \text{ in (I)} \qquad \text{or} \qquad \Lambda = \frac{N}{2} \ \text{ in (II)}.$$

We note that the blow-up result is remained open and the critical exponents are not yet determined when $b(t,x) = \langle x \rangle^{-\alpha}(1+t)^{-\beta} \quad (0 < \alpha + \beta < 1, \alpha > 0, \beta > 0)$.

This talk is based on the joint work with Yuta Wakasugi.