On t-dependent hyperbolic systems

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Abstract In the joint paper [1] with M. Ruzhansky we considered t-dependent hyperbolic systems and derived dispersive type estimates for time-dependent hyperbolic systems under rather generic assumptions for the full symbol of the operator within the hyperbolic zone. The assumptions concerning small frequencies (mild form of dissipativity) were rather artificial. In the light of the recent paper [2] with W. Nunes, we reformulate them in a more natural manner and consider the Cauchy problem

$$\mathrm{D}_t U = A(t, \mathrm{D}_x)U, \qquad U(0, \cdot) = U_0$$

for a vector valued function $U = U(t, x)$ having d components, $x \in \mathbb{R}^n$, $t \geq 0$. As usual $\mathrm{D} = -\mathrm{i}\partial$ denotes the Fourier derivative and conditions are imposed on the full symbol $A(t, \xi)$ of the Fourier multiplier $A(t, \mathrm{D}_x)$. There are two kinds of conditions involved,

- first: uniform strict hyperbolicity conditions on the large-frequency principal part accompanied by a technical condition implying a generalised energy conservation property for the hyperbolic zone;
- second: conditions involving a large-time, small frequency principal part

  $$tA(t, \xi) \sim A_p, \qquad t \to \infty, t|\xi| \ll 1,$$

  coming into play whenever the equation is close to be scale-invariant.

In combination, both conditions allow for an asymptotic construction of the fundamental solution of the problem and yield enough structural information to derive energy and dispersive type inequalities for its solutions.

Bibliography

[1] M. Ruzhansky and J. Wirth. Dispersive estimates for hyperbolic systems with time-dependent coefficients. *J. Differential Equations*, **251** (2011) 941–969.

[2] W. Nunes do Nascimento and J. Wirth. Wave equations with mass and dissipation. *Advances in Differential Equations*, **20** (2015) 661-696.