

**ABSTRACTS**  
THE 26TH MATSUYAMA CAMP  
RECENT DEVELOPMENT  
ON PARTIAL DIFFERENTIAL EQUATIONS

**Radjesvarane, ALEXANDRE**, (Shanghai Jiao Tong University)

**Lower and upper estimates for the non-cutoff collisional operator**

Following the talk of Seiji Ukai, we will discuss estimates for the collisional linearized and nonlinear operators, for non cutoff cross-sections. Among them, we will introduce a new coercive non-isotropic norm related to the linearized operator, and which gives a good control of the nonlinear ones. These estimates are among the most important tools involved and developed in the recent works for the Cauchy problem near equilibrium, made by R.A., Yoshinori Morimoto, Seiji Ukai, Chao-Jiang Xu and Tong Yang.

**Hua CHEN** (Wuhan University)

**Regularities of Solutions for a Class of Degenerate Partial Differential Equations**

**Setsuro FUJIE** (Ritsumeikan University)

**Semiclassical resonance width for homoclinic trajectories** (joint work with J.-F.Bony, T.Ramond, M.Zerzeri)

**Fumihiko HIROSAWA** (Yamaguchi University)

**On the energy estimates for second order homogeneous hyperbolic equations with Levi-type conditions**

**Toshihiko HOSHIRO** (Hyogo Prefecture University)

**Nonlinear eigenvalue problem and related topics**

In this talk, we make a survey of some recent improvements of nonlinear eigenvalue problem. This problem can be considered as a subject of non selfjoint spectral theory, and eigenvalues of operators with polynomial dependence are discussed. The original motivation to such a problem was related to dissipative problem in mechanics. Our motivation here is related to nonanalytic hypoellipticity. We hope to give a new approach to this problem.

**Haruhisa ISHIDA** (University of Electro-Communications)

**The Lyapunov exponents for nonhomogeneous linear differential systems**

**Chisato IWASAKI** (University of Hyogo)

**Construction of the fundamental solutions and spectral functions of nilmanifolds**

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**Yasunori MAEKAWA** (Kobe University)

## On fundamental solutions related to non-local Dirichlet forms with BMO convections

In this talk we consider some parabolic equations related to non-local Dirichlet forms with convection terms, which involves, as a special case, a linearized equation for two-dimensional dissipative quasi-geostrophic equations. We will establish pointwise upper bounds and continuity estimates for fundamental solutions of the equations. This talk is based on a joint work with Hideyuki Miura (Osaka University).

Andre MARTINEZ (University of Bologna)

### Life-time for the molecular predissociation (joint work with A. Grigis)

We consider a semiclassical  $2 \times 2$  matrix Schrödinger operator of the form  $P = -h^2 \Delta \mathbf{I}_2 + \text{diag}(V_1(x), V_2(x)) + hR(x, hD_x)$ , where  $V_1, V_2$  are real-analytic,  $V_2$  admits a non degenerate minimum at 0,  $V_1$  is non trapping at energy  $V_2(0) = 0$ , and  $R(x, hD_x) = (r_{j,k}(x, hD_x))_{1 \leq j,k \leq 2}$  is a symmetric off-diagonal  $2 \times 2$  matrix of first-order pseudodifferential operators with analytic symbols. We also assume that  $V_1(0) > 0$ . Then, denoting by  $e_1$  the first eigenvalue of  $-\Delta + \langle V_2''(0)x, x \rangle / 2$ , and under some ellipticity condition on  $r_{1,2} = r_{2,1}^*$ , we show that the unique resonance  $\rho$  of  $P$  such that  $\rho = V_2(0) + (e_1 + r_{2,2}(0, 0))h + \mathcal{O}(h^2)$  (as  $h \rightarrow 0_+$ ) satisfies,

$$\text{Im } \rho = -h^{(1-n_\Gamma)/2} f(h, \ln \frac{1}{h}) e^{-2S/h},$$

where  $f(h, \ln \frac{1}{h}) \sim \sum_{0 \leq m \leq \ell} f_{\ell,m} h^\ell (\ln \frac{1}{h})^m$  is a symbol with  $f_{0,0} > 0$ ,  $S > 0$  is the so-called Agmon distance associated with the degenerate metric  $\max(0, \min(V_1, V_2)) dx^2$ , between 0 and  $\{V_1 \leq 0\}$ , and  $n_\Gamma$  is a positive integer.

Toshiyuki MIYOSHI (Ryukoku University, Master Course)

### Blow up Phenomena on the curvature of closed plane elastic curves

Yoshinori MORIMOTO (Kyoto University)

### Uniqueness of solutions for non-cutoff Boltzmann equation and singular change of variables in pres-post collisional velocities

In this talk we consider the uniqueness of solution to the Cauchy problem for non-cutoff Boltzmann equation in the whole space. Several results in different function spaces are detailed in the cases of hard and soft potentials. In particular, we discuss the uniqueness of the solution with the polynomial decay with respect to the velocity variable, in the soft potential case of the classical sense, where the singular change of variables from “pres” to “post” collisional velocity plays an important role. The content of this talk is a part of the joint-works with R. Alexandre, S. Ukai, C.-J. Xu and T. Yang.

Minoru MURAI (Ryukoku University, Post-Doctor)

### Blow up Phenomena on the curvature of closed plane elastic curves with the winding number $\omega \geq 1$

Tatsuo NISHITANI (Osaka University)

### Note on lower bounds of energy growth for solutions to wave equation

Karel PRAVDA-STAROV (Chaire CNRS de Physique-Mathématique Université de Cergy-Pontoise)

## Optimal Hypoelliptic Estimates for a Class of Kinetic Equations

In this talk, we shall discuss the hypoelliptic properties of a class of kinetic equations, which are linear models of spatially inhomogeneous Boltzmann equations without angular cutoff. More specifically, we shall study the hypoelliptic properties of the kinetic operator

$$P = \partial_t + v \cdot \partial_x + a(t, x, v)(-\tilde{\Delta}_v)^\sigma, \quad t \in \mathbb{R}, \quad x, v \in \mathbb{R}^n;$$

where  $0 < \sigma < 1$  is a constant parameter and  $a$  denotes a positive  $C^\infty(\mathbb{R}^{2n+1})$  function

$$a(t, x, v) > 0, \quad (t, x, v) \in \mathbb{R}^{2n+1}.$$

Here  $(-\tilde{\Delta}_v)^\sigma$  stands for the Fourier multiplier of symbol

$$F(\eta) = |\eta|^{2\sigma}w(\eta) + |\eta|^2(1 - w(\eta)), \quad \eta \in \mathbb{R}^n;$$

with  $|\cdot|$  being the Euclidean norm and  $w \in C^\infty(\mathbb{R}^n)$ ,  $0 \leq w \leq 1$ ,  $w(\eta) = 1$  if  $|\eta| \geq 2$ ; and  $w(\eta) = 0$  if  $|\eta| \leq 1$ . When  $\sigma = 1$ , this operator reduces to the so-called Vlasov-Fokker-Planck operator; whereas when  $0 < \sigma < 1$ , it stands for a linear model of the spatially inhomogeneous Boltzmann equation without angular cutoff. This is here the main motivation for studying the regularizing properties of this linear model and establishing hypoelliptic estimates with optimal loss of derivatives with respect to the exponent  $0 < \sigma < 1$  of the fractional Laplacian  $(-\tilde{\Delta}_v)^\sigma$ .

This is a joint work with Prof. Nicolas Lerner and Prof. Yoshinori Morimoto.

**Vania SORDONI** (University of Bologna)

### Stark effect on $H_2^+$ -like molecules

We consider the vibrational energy levels of the first two electronic states of the molecule ion  $H_2^+$ . The Born-Oppenheimer method applied to the case of the Stark effect on a  $H_2^+$ -like molecule gives existence of sharp resonances localized in the same interval of energy of the vibrational levels.

**Hideki TAKUWA** (Doshisha University)

### The construction of a class of solutions to hyperbolic equations and applications to inverse problems

**Seiji UKAI** (Professor Emeritus Tokyo Institute of Technology)

### Boltzmann equation and the existence of solutions

After presenting a brief summary on the existence theories developed so far for the Boltzmann equation, we give an overview of the recent progress on the non-cutoff case achieved by a series of joint works by R. Alexandre, Y. Morimoto, C.-j. Xu, T. Yang, and myself. Open problems for the future study are also discussed.

**Seiichiro WAKABAYASHI** (University of Tsukuba)

### Singularities of solutions to the Cauchy problem for second-order hyperbolic operators with the coefficients of their principal parts depending only on the time variable

**Chao-Jiang, XU** (Wuhan University/Université de Rouen)

### Regularity of solutions for the non-cutoff Boltzmann equation

In this talk, we establish the hypoellipticity of the inhomogeneous Boltzmann equation without angular cutoff. By using the nonlinear microlocal analysis, we can study this equation as a generalized Kolmogorov equation which is non linear and non-isotropic

equation. The key step to obtain the regularizing effect is a generalized version of the uncertainty principle, it is a very strong results of microlocal analysis, from which we prove the hypoellipticity of a transport equation. We present also the regularizing effect results in Gevrey class and analytic function spaces.

**Tong YANG** (City University of Hong-Kong)

### **Hypocoercivity in Kinetic Theory and the Convergence to the Equilibrium**

It is known that the coupling of some degenerate dissipative operator and a conservative operator gives full dissipation and convergence to the equilibrium. Based on the estimates on the collision operators for the Boltzmann equation without angular cutoff, we obtain the convergence rate estimates on solutions to the equilibrium for both soft and hard potentials. Our approach combines the compensating function method introduced by Kawashima for the Boltzmann equation and the energy method. The results of this presentation is based on the joint work with R. Alexandre, Y. Morimoto, S. Ukai and C.-J. Xu.

**Claude Zuily** (Professor Emeritus Université de Paris XI )

### **On the Cauchy problem for the water wave system**