# International Workshop

# "Fourier Analysis and Partial Differential Equations"

## — Program and Abstracts —

University of Göttingen

June 14–17, 2010

# Program

| | **June 14 Mon** | **June 15 Tue** | **June 16 Wed** | **June 17 Thu** |
|---|---|---|---|---|
| *8:45–9:00* | *Opening* | | | |
| *Chair* | *Witt* | *Iwasaki* | *Sugimoto* | *Ruzhansky* |
| 9:00–9:40 | Ozawa | Gramchev | Aripov | Pilipovic |
| 9:50–10:30 | d'Ancona | Bahns | Schrohe | Gröchenig |
| | **Coffee break** | | | |
| *Chair* | *Gröchenig* | *Georgiev* | *Schrohe* | *Wang* |
| 11:00–11:40 | Wang | Rodino | Bauer | Toft |
| 11:50–12:30 | Georgiev | Garetto | Seiler | Dreher |
| | **Lunch** | | | |
| *Chair* | *Toft* | *d'Ancona* | *Pilipovic* | *Rodino* |
| 15:00–15:40 | Tomita | Hirosawa | Kalmenov | Kobayashi |
| 15:50–16:30 | Turunen | Wirth | Nguyen | Yamane |
| | **Coffee break** | | | |
| *Chair* | *Kalmenov* | *Ozawa* | *Matsuyama* | |
| 17:00–17:40 | Bez | Böhme | Kmit | Schulze (Colloquium, 17:15 – 18:15) |
| 17:50–18:30 | Chiba | Matsuyama | Iwasaki | |

# Abstracts

**Mirsaid Aripov** (Tashkent)

*The Cauchy problem for a reaction-diffusion equation with a double nonlinearity in a strong absorption case*

**Abstract:** Consider the Cauchy problem

$$Au \equiv \frac{\partial u}{\partial t} - L(m,p)u + \operatorname{div}(v(t)u) - u^{\beta} = 0 \tag{1}$$

$$u|_{t=0} = u_0(x) \geqslant 0, \quad x \in \mathbb{R}^N, \tag{2}$$

where $L(m,p)u = \nabla(u^{m-1}\,|\nabla u|^{p-2}\,\nabla u)$, $m > 1$, $p \geqslant 2$, $0 < \beta < 1$ are given numerical parameters, $\nabla(\cdot) = \operatorname{grad}_x(\cdot)$, $0 < \gamma(t) \in C(0,\infty)$, $\varepsilon = \pm 1$, and $0 < v(t)$ is a matrix function.

The possibilities of self-similar and approximately self-similar approaches to studying the qualitative properties of the Cauchy problem for the reaction-diffusion equation with double nonlinearity in a strong absorption case based on the method of nonlinear splitting is demonstrated. Global solvability, estimates of weak solutions, and the free boundary problem for the Cauchy problem are considered , in the critical and double critical cases. It is proved that there exist certain values of the parameters when the effects of finite velocity of perturbations, localization of solution, and onside localization have place. Based on established qualitative properties of solution the numerical computations are carried out.

**Theorem.** *The following condition on global solvability* $3 < m + p < 4$, $N \geqslant 3$, $4 - (m+p) - \frac{p}{N} < \beta < 4 - (m+p)$ *to solution of problem* (1)-(2) *for a small initial data is valid.*

In the critical case when $m + p = 3$ we have a condition on global solvability $1 - \frac{p}{N} < \beta < 1$, $N \geqslant 3$.

The critical exponential case is $m+p > 3$, $\beta = 4-(m+p) - \frac{p}{N}$ and the double critical case is $m + p = 3$, $\beta = 1 - \frac{p}{N}$. It is shown that the behavior of solutions in these cases changes.

(Joint work with Sh. Sadullaeva.)

**Dorothea Bahns** (Göttingen)

*Gelfand-Shilov functions in quantum field theory on Moyal space*

**Abstract:** The correct choice of test functions in perturbative quantum field theory on the noncommutative Moyal space is a subtle question. I will give some indications why Gelfand-Shilov functions (functions of type $S$) provide a good framework for the formulation of the renormalization problem in this setting.

**Wolfram Bauer** (Göttingen)

*Heat flow, BMO and the compactness of Toeplitz operators*

**Abstract:** It is well-known that there are close relations between the Berezin Toeplitz calculus on the Segal-Bargmann space, pseudo-differential operators in Weyl quantization and the heat transform. In this talk we describe some aspects of this interplay. In particular, we focus on the following problem: for functions on $\mathbb{C}^n$ of bounded mean oscillation we can take the Berezin transform (or *heat transform*) $\widetilde{f}^{(t_0)}$ of $f$ with respect to the weighted Segal-Bargmann space $H^2(\mathbb{C}^n, d\mu_{t_0})$ where $t_0$ can be interpreted as a time parameter. We discuss the question whether $\widetilde{f}^{(t_0)}$ vanishing at infinity for some $t_0 > 0$ implies that $\widetilde{f}^{(t)}$ vanishes at infinity for all "times" $t > 0$. This is joint work with L.A. Coburn and J. Isralowitz. On the one hand, this analysis of the heat transform uses results in operator theory. On the other hand, by solving the above problem we can give compactness and Schatten-p-class characterizations of Toeplitz operators. We indicate that these results fail in case of arbitrary unbounded symbols. Finally, a similar question can be discussed in the context of weighted Bergman spaces over the unit ball or more generally over bounded symmetric domains. However, there seems to be no natural relation to the heat transform.

**Neal Bez** (Birmingham)

*Some sharp Sobolev-Strichartz estimates*

**Abstract:** For initial data in certain Sobolev spaces it is known that maximisers for Strichartz space-time estimates for the Schrödinger and wave equations exist. We shall discuss the problem of locating these maximisers and the optimal constants in these estimates.

**Christiane Böhme** (Freiberg)

*$L^p$-$L^q$ decay estimates for a scale-invariant Klein-Gordon equation*

**Abstract:** In this talk we investigate scale-invariant Klein-Gordon type models given by

$$u_{tt} - \triangle u + m^2(1+t)^{-2}u = 0,$$

$\triangle = \sum_{j=1}^{n} \partial_{x_j}^2$, with constant $m$. Using the theory of special functions we can prove that the $L^p$-$L^q$ decay estimates are of wave type. Thus, they describe a particular situation within the consideration of Klein-Gordon equations.

**Yasuo Chiba** (Tokyo)

*Examples of hyperbolic equations and their reductions by a quantized Legendre transform*

**Abstract:** Some weakly hyperbolic operators can be reduced to the microdifferential operators by a quantized contact transform. We will show the process of the reduction. In particular, we introduce typical examples of operators of Fuchs type.

**Piero d'Ancona** (Rome)

*Smoothing and Strichartz estimates on deformed waveguides*

**Abstract:** We investigate the dispersive properties of evolution equations in waveguides which are compactly supported perturbations of cylindrical sets. We obtain in particular a smoothing estimate and Strichartz estimates with loss of derivatives for the Schrödinger equation on such domains.

(Joint work with R. Racke, Konstanz.)

**Michael Dreher** (Konstanz)

*Douglis-Nirenberg systems and analytic semigroups*

**Abstract:** We study mixed order parameter-elliptic boundary value problems with boundary conditions of a certain structure. For such operators, we prove the analyticity of the semigroup via resolvent estimates in $L^p$ based Sobolev spaces of suitable order; and we present an application of this theory to studies of the particle transport in a semi-conductor.

**Claudia Garetto** (London)

*Generalised functions and hyperbolic operators with singular coefficients*

**Abstract:** In this talk we present a generalised function approach to hyperbolic problems with singular coefficients. A particular attention is given to the notion of generalised strict hyperbolicity.

**Vladimir Georgiev** (Pisa)

*Resonances and local energy decay near solitary solutions*

**Abstract:** It is well-known that asymptotic stability around solitary waves is closely connected with the existence of resonances at the origin. More precisely, the following assumption is frequently used:

(H1) $\qquad 0$ is not a resonance of $-\triangle - p\chi_*^{p-1}(x)$,

where $\chi$ satisfies the equation

$$-\Delta\chi + \omega\chi = |\chi|^{p-1}\chi, \quad x \in \mathbb{R}^3,$$

and $\int_{\mathbb{R}^3} \chi^2 = 1$.

The main goal of this work is to present an argument that proves the assumption (H1) in the radially symmetric case and therefore the stability results can be established without this additional assumption when radially symmetric solutions are considered.

The result is applied to establish the exponential local energy decay for the corresponding evolution problems.

(Joint work with Mirko Tarulli, Pisa.)

**Todor Gramchev** (Cagliari)

*Decay and global regularity for solutions of pseudodifferential equations in $\mathbb{R}^n$*

**Abstract:** The main goal of the talk is to outline an abstract approach, based on techniques from microlocal analysis and perturbative methods in weighted Sobolev spaces and Gelfand-Shilov spaces, for the study of the decay for $|x| \to \infty$ and the regularity in $\mathbb{R}^n$ of solutions of (semi-)linear (pseudo-)differential equations in $\mathbb{R}^n$.

(Joint work with M. Cappiello and L. Rodino, Università di Torino.)

**Karlheinz Gröchenig** (Vienna)

*The range of localization operators and lifting properties of modulation spaces*

**Abstract:** Time-frequency localization operators arise in various applications and in mathematical theory: time-frequency masking in signal processing, phase-space localization (Daubechies), Toeplitz operators on Bargmann-Fock space (Berger-Coburn), quantization (Berezin), proofs of the Gårding and Fefferman-Phong inequalities (Lerner).

We will address one of the open problems, namely the question of how to understand the range of a time-frequency localization operator. Precisely, a localization operator is defined as follows: for a point $z = (x, \xi)$ in phase-space $\mathbb{R}^{2d}$ define the phase-space shift $\pi(z)$ acting on a function $f$ as

$$\pi(z)f(t) = e^{2\pi i \xi \cdot t} f(t - x) \qquad x, \xi, t \in \mathbb{R}^d \,.$$

Next fix a suitable "window function" $g$ in $\mathcal{S}(\mathbb{R}^d)$, say, and a "symbol" $m(z)$ on $\mathbb{R}^{2d}$. Then the localization operator $A^g_m$ is defined formally by

$$A^g_m f = \int_{\mathbb{R}^{2d}} m(z) \langle f, \pi(z)g \rangle \pi(z) g \, dz$$

The definition resembles the definition of Fourier multipliers, except that in the case of localization operators it is the short-time Fourier transform $\langle f, \pi(z)g \rangle$ that is multiplied by the symbol $m$ before the inverse transform is applied.

The relevant function spaces for the study of mapping properties of localization operators are the modulation spaces (as always in time-frequency analysis). For a non-zero test function $g$, usually the Gaussian, the modulation space $M^{p,q}_m$ is defined by the norm

$$\|f\|_{M^{p,q}_\mu} = \Big( \int_{\mathbb{R}^d} \Big( \int_{\mathbb{R}^d} |\langle f, \pi(x,\xi)g \rangle|^p \mu(x,\xi)^p \, dx \Big)^{q/p} \, d\xi \Big)^{1/q}$$

for $1 \leq p, q \leq \infty$ and moderate weight functions $\mu$, quite in analogy to the classical Besov spaces.

We will discuss results of the following type, so-called isomorphism theorems.

**Theorem.** *If $m$ is a moderate weight function, then $A^g_m$ is an isomorphism from $M^{p,q}_\mu$ **onto** $M^{p,q}_{\mu/m}$ for every $1 \leq p, q \leq \infty$ and moderate weight $\mu$.*

The isomorphism theorem resembles the lifting theorems for Besov spaces, where the operator of fractional differentiation establishes an isomorphism between Besov spaces of different smoothness. Modulation spaces can be interpreted as function spaces that describe the smoothness through the phase space distribution. In this sense the isomorphism theorem for localization operators is a lifting theorem

for modulation spaces. [Of course, the actual statement of the isomorphism theorem requires precise conditions on the window $g$ and on the symbol $m$.]

For weights of polynomial growth or decay the isomorphism theorem has been established recently with J. Toft. The techniques used pseudodifferential calculus and a deep result of Bony and Chemin.

These methods fail for super-algebraic weights, for instance when $m(z) = e^{a|z|^b}$ for $a \in \mathbb{R} \setminus \{0\}$ and $0 < b < 1$ or $m(z) = e^{\frac{|z|}{\log(e+|z|)}}$. To treat symbols with a growth or decay faster than polynomial, we develop a new technique based on pure time-frequency methods. The main aspects are (a) the spectral invariance of pseudodifferential operators in the generalized Sjöstrand class $M^{\infty,1}_v(\mathbb{R}^{2d})$, and (b) the explicit construction of isomorphisms between the $L^2(\mathbb{R}^d)$ and the modulation space $M^2_m(\mathbb{R}^d)$. It is a bizarre fact that the surjectivity of certain localization operators follows from a new class of inequalities about generalized Gamma functions labeled by moderate weight function $\theta$. As a example we mention the inequality

$$C^{-1} \leq \Big( \int_0^\infty \theta(\sqrt{x/\pi}) \frac{x^n}{n!} e^{-x} dx \Big)^2 \int_0^\infty \frac{1}{\theta(\sqrt{x/\pi})^2} \frac{x^n}{n!} e^{-x} dx \leq C, \quad \forall n \geq 0.$$

The obtained results yield also new insights into Toeplitz operators on the Bargmann-Fock space of entire functions on $\mathbb{C}^d$ and on Gabor multipliers (earlier considered by Feichtinger).

(Joint work with Joachim Toft, Linnäus University of Växjö.)

**Fumihiko Hirosawa** (Yamaguchi)

*On the energy estimates of second order hyperbolic equations with time dependent coefficients*

**Abstract:** We consider the energy estimates for second order homogeneous hyperbolic equations with time dependent coefficients. If the coefficients are constants, then the energy conservation is valid; but it is not true for variable coefficients case. If only one of the coefficients is variable, then we have studied that the sufficient conditions for the generalized energy conservation (=GEC), which is an energy conservation in a general meaning, are given by the oscillation and the smoothness properties of the coefficient. However, if more than two of the coefficients are variable, then the same conditions to the coefficients as the single variable coefficient case cannot conclude GEC, because some resonance phenomenon of these coefficients are possible to make the energy unstable. In my talk we introduce some examples of the coefficients, which conclude GEC and non-GEC.

(Joint work with Bui Tang Bao Ngoc from Hanoi University of Technology, Vietnam.)

**Chisato Iwasaki** (Himeji)

*A representation of the fundamental solution to some heat equation and its applications*

**Abstract:** I will show how to construct an exact form of the fundamental solution to heat equations of polynomial coefficients. The fundamental solution is obtained as a pseudo-differential operators. I will give applications of this exact form.

(Joint work with W. Bauer and K. Furutani.)

**Tynysbek Kalmenov** (Almaty)

*The regular Vollterov boundary value problem for the Laplace equation*

**Irina Kmit** (Berlin)

*Fredholmness and smooth dependence for linear hyperbolic periodic-Dirichlet problems*

**Abstract:** The talk is devoted to $n \times n$ linear one-dimensional hyperbolic systems of the type

$$\omega \partial_t u_j + a_j(x) \partial_x u_j + \sum_{k=1}^{n} b_{jk}(x) u_k = f_j(x,t), \quad j = 1, \dots, n,$$

with periodicity conditions in time and reflection boundary conditions in space. We state sufficient conditions on the data $\omega$, $a_j$ and $b_{jk}$ such that the system has a Fredholm like solvability behavior. Moreover, we state sufficient conditions on the data such that for any right hand side there exists exactly one solution, that the solution survives under small perturbations of the data, and that the corresponding data-to-solution-map is smooth with respect to appropriate function space norms. In particular, these conditions rule out small denominator effects. We also show that perturbations of the coefficients $\omega$ and $a_j$ make essentially different effects than perturbations of the coefficients $b_{jk}$.

(Joint work with Lutz Recke, HU Berlin)

**Masaharu Kobayashi** (Tokyo)

*Schatten $p$-class property of pseudo-differential operators*

**Abstract:** We shall consider sufficient conditions on symbols $\sigma(x,\omega)$ to ensure the operators $\sigma(X,D)$ and $\sigma^W(X,D)$ to belong to the Schatten $p$-class $C_p$, $0<p\leq 2$.

(Joint work with Akihiko Miyachi, Tokyo Woman's Christian University.)

**Tokio Matsuyama** (Hiratsuka)

*Dispersion for hyperbolic equations with variable coefficients*

**Abstract:** We will review the recent results on the dispersive and Strichartz estimates for hyperbolic equations with variable coefficients involving strictly hyperbolic equations of higher order with time-dependent coefficients and wave equation with a potential. These estimates for hyperbolic equations with constant coefficients are rather well known from many results. In this sense the topics of this talk will provide a new aspect of hyperbolic equations.

**Manh Hung Nguyen** (Hanoi)

*On the regularity of the solutions for the first initial boundary value problem for hyperbolic systems in infinite cylinder with non-smooth base*

**Abstract:** The purpose of this paper is to establish the regularity of generalized solutions of the first initial boundary value problem for hyperbolic systems in infinite cylinders with the non-smooth base. Some results on the solvability of this problem are given.

**Tohru Ozawa** (Tokyo)

*Remarks on some dispersive estimates*

**Abstract:** We consider a general dispersive equation, including Schrödinger and Klein-Gordon types. We prove the corresponding dispersive and Strichartz estimates.

(Joint work with Yonggeun Cho.)

**Stevan Pilipovic** (Novi Sad)

*Microlocal analysis through Tauberian type characterizations of spaces of functions and generalized functions*

**Abstract:** We will present the notions of generalized asymptotic behaviour and

use quasiasymptotic boundedness for the characterization of the wavelet transform within various spaces. Abelian type results will be illustrated by some classical results. Then we present Tauberian type results for vector valued distributions and apply them to the characterizations of local and microlocal properties within spaces of functions, distributions and generalized function algebras.

(Joint work with Dr Jasson Vindas.)

**Luigi Rodino** (Torino)

*Global regularity for ordinary differential operators with polynomial coefficients*

**Abstract:** We consider an ordinary differential operator $P$ with polynomial coefficients. We say that $P$ is globally regular if, for every distribution $u$ in the Schwartz space $S'$, the $S$-regularity of $Pu$ implies the $S$-regularity of $u$. In particular, when $P$ is globally regular, all the solutions $u$ in $S'$ of the homogeneous equation $Pu = 0$ belong to $S$. We give a necessary and sufficient condition for the global regularity of $P$.

**Elmar Schrohe** (Hannover)

*Regularity of the eta function for boundary value problems*

**Abstract:** On a compact manifold $X$ with boundary we consider the realization $B = P_T$ of an elliptic boundary problem, consisting of a differential operator $P$ of positive order and a differential boundary condition $T$.

We assume that $B$ is parameter-elliptic in small sectors around two rays in the complex plane, say $\arg\lambda = \phi$ and $\arg\lambda = \theta$. Associated to the cuts along the rays one can then define two zeta functions $\zeta_\phi$ and $\zeta_\theta$ for $B$. Both extend to meromorphic functions on the plane; the origin is a regular point. In the boundaryless case, a deep result, going back to Atiyah–Patodi–Singer, Gilkey and Wodzicki, shows that $\zeta_\phi(0) = \zeta_\theta(0)$.

Interest in this question arose from the following observation: When $B$ is self-adjoint, and one ray lies in the upper half plane and one in the lower, then $\zeta_\phi(0) - \zeta_\theta(0)$ equals – up to a constant – the residue of the eta function at the origin. The fact that the zeta values coincide thus implies the regularity of the eta function. In the boundaryless case this is crucial for the Atiyah-Patodi-Singer index theorem.

In the case of boundary value problems progress was made recently by Grubb and Gaarde. Following the approach taken by Wodzicki, they related the difference of the values at the origin to the associated sectorial projection $\Pi_{\theta,\phi}(B)$ defined by

$$\Pi_{\theta,\phi}u = \frac{i}{2\pi}\int_{\Gamma_{\theta,\phi}} \lambda^{-1}B(B-\lambda)^{-1}u\,d\lambda, \quad u \in \mathrm{dom}(B).$$

Here $\Gamma_{\theta,\phi}$ is a contour which runs on the first ray from infinity to $r_0 e^{i\phi}$ for some $r_0 > 0$, then clockwise about the origin on the circle of radius $r_0$ to $r_0 e^{i\theta}$ and back to infinity along the second ray.

Grubb and Gaarde showed that whenever $\Pi_{\theta,\phi}$ is an operator of order and class zero in Boutet de Monvel's calculus, its noncommutative residue (in the sense of Fedosov-Golse-Leichtnam-Schrohe (FGLS)) coincides with the difference of the zeta functions in zero. Moreover, Gaarde proved that the noncommutative residue then is zero, so that the values of the zeta functions agree.

In most cases, however, the sectorial projection will not belong to the Boutet de Monvel algebra, and the above methods are not applicable.

We solve the problem by constructing an operator algebra which extends Boutet de Monvel's. It is large enough to contain the sectorial projections of the above boundary value problems, yet its K-theory naturally coincides with that of Boutet de Monvel's algebra. Moreover, we show that it carries a noncommutative residue which extends that of FGLS and vanishes on projections. Hence the values of the zeta functions will coincide and, if $B$ is self-adjoint, its eta function will be regular in zero.

(Joint work with H. Gimperlein, Hannover.)

**Bert-Wolfgang Schulze** (Potsdam)

*Ellipticity and asymptotics on manifolds with geometric singularities*

**Abstract:** The analysis of elliptic (and other types of) equations on a manifold with singularities is motivated by applications when a model is affected by a singular geometry of the underlying configuration (e.g. of conical or edge type), or by singularities of the coefficients in a corresponding partial differential equation. Even relatively simple equations, containing Laplacians plus singular potentials, or boundary conditions with discontinuous data (such as a jump from Dirichlet to Neumann conditions) induce interesting structures in the context of parametrices and regularity of solutions. For instance, when we start the solvability discussion for a single elliptic operator in the case of an edge singularity (e.g. a boundary), there arise operator-valued symbols, families of meromorphic amplitude functions, topological obstructions for the existence of elliptic edge conditions, quantisation procedures, operator algebras, symbolic hierarchies, homotopies of the operators together with the underlying spaces, index problems, variable and branching asymptotic phenomena, and many new (partly unexpected) challenges. All this belongs to the recent development of the analysis on singular manifolds. We give an overview on the state of this theory, and we present a new approach in terms of hierarchies of stratified spaces and an iterative process of building up algebras of "higher" pseudo-differential operators, closed under the construction of parametrices of elliptic ele-

ments.

**Jörg Seiler** (Loughborough)

*On the noncommutative residue for projective pseudodifferential operators*

**Abstract:** The noncommutative residue (Wodzicki residue) of a pseudodifferential projection is known to be zero. This statement is non-local and implies the regularity of the eta invariant at zero of Dirac type operators. We prove that in a filtered algebra the value of a projection under any residual trace depends only on the principal part of the projection. This general, purely algebraic statement applied to the algebra of projective pseudodifferential operators implies that the noncommutative residue factors to a map from the twisted K-theory of the co-sphere bundle. We show that this map vanishes in odd dimensions, thus showing that the noncommutative residue of a projective pseudodifferential projection vanishes.

(Joint work with A. Strohmaier, Loughborough.)

**Joachim Toft** (Växjö)

*Global wave-front sets of Fréchet types*

**Abstract:** Roughly speaking, the local wave-front set $\mathrm{WF}^{\psi}_{\mathcal{B}}(f)$ of the distribution $f$ with respect to the Banach or Fréchet space $\mathcal{B}$, gives information *where* the distribution $f$ has singularities with respect to $\mathcal{B}$, as well as *what directions* in these points of singularities, the singularities propagates.

In this talk we present a triple $(\mathrm{WF}^{\psi}_{\mathcal{B}}(f), \mathrm{WF}^{e}_{\mathcal{B}}(f), \mathrm{WF}^{\psi e}_{\mathcal{B}}(f))$ of wave-front sets. Here the component $\mathrm{WF}^{e}_{\mathcal{B}}(f)$ informs what directions, the size of $f$ near infinity is large, and $\mathrm{WF}^{\psi e}_{\mathcal{B}}(f))$ informs what directions $f$ has strong oscillations near infinity, everything with respect to $\mathcal{B}$. An important large class of admissible Fréchet spaces is the family of modulation spaces.

An advantage with the local component of such wave-front sets comparing to wave-front set of smoothness (i.e., wave-front sets with respect to $C^{\infty}$), is that we may examine micro-local properties which are more close to differentiability up to a certain degree, instead of infinitely differentiability. Furthermore, the global components $\mathrm{WF}^{e}_{\mathcal{B}}(f)$ and $\mathrm{WF}^{\psi e}_{\mathcal{B}}(f))$ give detailed information of $f$ far away from the origin.

An important property that our wave-front sets fulfill concerns the embedding

$$\mathrm{WF}^{\psi}_{\mathcal{C}}(\mathrm{Op}(a)f) \subseteq \mathrm{WF}^{\psi}_{\mathcal{B}}(f) \subseteq \mathrm{WF}^{\psi}_{\mathcal{C}}(\mathrm{Op}(a)f) \bigcup \mathrm{Char}(a),$$

and similar for the other components. Here $\mathcal{C}$ is closely related to $\mathcal{B}$, $\mathrm{Op}(a)$ is a partial differential operator with smooth coefficients or a pseudo-differential operator,

and $\mathrm{Char}(a)$ is the set of characteristic points for $a$ with respect to $\omega$.

In the talk we give some links on how to prove such relations, and we present some consequences on global hypoellipticity.

(Joint work with with Sandro Coriasco, Karoline Johansson, Stevan Pilipovic and Nenad Teofanov.)

**Naohito Tomita** (Osaka)

*A Hörmander type multiplier theorem for multilinear operators*

**Abstract:** In this talk, we consider a Hörmander type multiplier theorem for multilinear operators. As a result, we can weaken the regularity assumption for multilinear Fourier multipliers to assure the boundedness.

**Ville Turunen** (Aalto)

*Sharp Gårding inequality on compact Lie groups*

**Abstract:** We establish the sharp Gårding inequality on compact Lie groups. The symbol positivity condition is expressed in the non-commutative phase space in terms of the full global symbol, which is defined using the representations of the group. An application is given to the $L^2$-boundedness of pseudo-differential operators.

(Joint work with M. Ruzhansky, Imperial College London.)

**Baoxiang Wang** (Beijing)

*Inviscid limit for the derivative Ginzburg-Landau equation with small data in higher spatial dimensions*

**Abstract:** We study the Cauchy problem for derivative Ginzburg-Landau equation

$$u_t = (\nu + i)\triangle u + \overrightarrow{\lambda_1} \cdot \nabla(|u|^2 u) + (\overrightarrow{\lambda_2} \cdot \nabla u)|u|^2 + \alpha |u|^{2\delta} u,$$

where $\delta \in \mathbb{N}$, $\overrightarrow{\lambda_1}, \overrightarrow{\lambda_2}$ are complex constant vectors, $\nu \in [0, 1]$, $\alpha \in \mathbb{C}$.

For $n \geq 3$, we show that it is uniformly global well-posed for all $\nu \in [0, 1]$ if initial data $u_0$ belong to modulation space $M^s_{2,1}$ ($s > 3$) and $\|u_0\|_{L^2} \ll 1$. Moreover, we show that its solution will converge to that of the derivative Schrödinger equation in $C(0, T; L^2)$ if $\nu \to 0$ and $u_0 \in M^4_{2,1}$. For $n = 2$, we obtain the local well-posedness results and inviscid limit with the Cauchy data in $M^s_{1,1}$ ($s > 3$) and

$\|u_0\|_{L^1} \ll 1$. The techniqie used in this paper are frequency-uniform decompositon, Strichartz and smooth effect estimates for the Ginzburg-Landau semigroup.

(Joint work with Dr Han Lijia and Professor Boling Guo.)

**Jens Wirth** (London)

*TBA*

**Hideshi Yamane** (Sanda)

*Logarithmic singularities of solutions to nonlinear partial differential equations*

**Abstract:** We construct a family of singular solutions to some nonlinear partial differential equations which have resonances in the sense of a paper due to T. Kobayashi. The leading term of a solution in our family contains a logarithm, possibly multiplied by a monomial. As an application, we study nonlinear wave equations with quadratic nonlinearities. The proof is done by the reduction to a Fuchsian equation with singular coefficients.